ELASTICITY & VISCOSITY

ELASTICITY AND PLASTICITY

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming force : An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic body : A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fiber is the nearest approach to the perfectly elastic body.

Perfectly Plastic body : A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, so the concept of perfectly plastic body is also only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity : In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighboring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are minimum.
On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

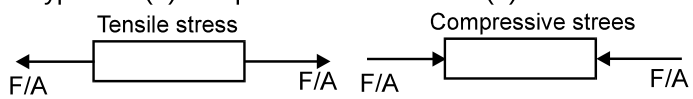
When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body is called stress.

$$\text{stress} = \frac{\text{restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2. There are three types of stress

1. Longitudinal or Normal stress

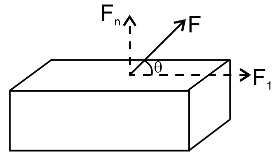
When object is one dimensional then force acting per unit area is called longitudinal stress.
It is of two types : (a) compressive stress (b) tensile stress



Examples :

(i) Consider a block of solid as shown in figure. Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components :
$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.

$\therefore$ Normal (tensile) stress $= \dfrac{F_n}{A} = \dfrac{F\sin\theta}{A}$



(b)

2. Tangential or shear stress

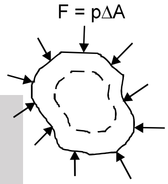
It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to shown in figure above.

Tangential (shear) stress = $\frac{F_t}{A} = \frac{F\cos\theta}{A}$

The effect of stress is to produce distortion or a change in size, volume and shape (i.e., configuration of the body).

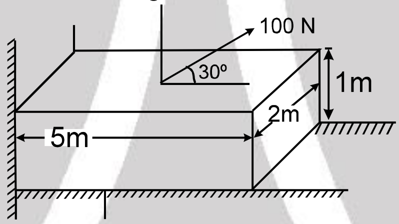
3. Bulk stress

When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon the homogeneity of body.



Solved Example

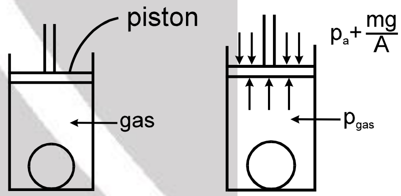
Example 1. Find out longitudinal stress and tangential stress on a fixed block.



Solution : Longitudinal or normal stress $\Rightarrow$ $\sigma_l = \frac{100\sin 30^o}{5\times 2}$ = 5 N/m^2

Tangential stress $\Rightarrow$ $\sigma_t = \frac{100\ \cos\ 30^o}{5\times 2} = 5\sqrt{3}\text{N/m}^2$

Example 2. Find out Bulk stress on the spherical object of radius $\frac{10}{\pi}$ cm if area and mass of piston is 50 cm^2 and 50 kg respectively for a cylinder filled with gas.



Solution : $p_{gas} = \frac{mg}{A} + p_a = \frac{50\times 10}{50\times 10^{-4}}$ + 1 × 10^5 = 2 × 10^5 N/m^2

Bulk stress = p_{gas} = 2 × 10^5 N/m^2

STRAIN

The ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body is called strain,

i.e. Strain, $\in = \frac{\text{change in configuration}}{\text{original configuration}}$

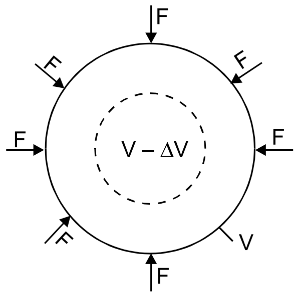
It has no unit

Types of strain : There are three types of strain

(i) Longitudinal strain : This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body. Consider a wire of length L **:** When the wire is stretched by a force F, then let the change in length of the wire is ΔL.

$\therefore$ Longitudinal strain, $\in_\ell = \frac{\text{change in length}}{\text{original length}}$ or Longitudinal strain = $\frac{\Delta L}{L}$

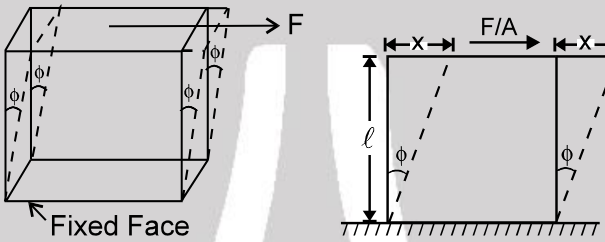
(ii) Volume strain : This type of strain is produced when the deforming force produces a change in volume of the body as shown in the figure. It is defined as the ratio of the change in volume to the original volume of the body.



If ΔV = change in volume V = original volume

$\in_V$ = volume strain = $\frac{\Delta V}{V}$

(iii) Shear Strain : This type of strain is produced when the deforming force causes a change in the shape of the body. It is defined as the angle ϕ through which a face originally perpendicular to the fixed face is turned as shown in the figure.



$\tan\phi$ or $\phi = \frac{x}{\ell}$

HOOKE'S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain

or stress = constant × strain or $\frac{\text{stress}}{\text{strain}}$ = Modulus of Elasticity.

This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

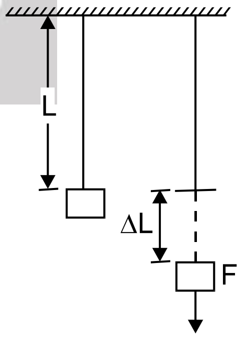
Modulus of elasticity depends on the nature of the material of the body and is independent of its dimensions (i.e. length, volume etc.).

Unit : The Sl unit of modulus of elasticity is Nm^{-2} or Pascal (Pa).

TYPES OF MODULUS OF ELASTICITY

Corresponding to the three types of strain there are three types of modulus of elasticity.

1. Young's modulus of elasticity (Y)
2. Bulk modulus of elasticity (K)
3. Modulus of rigidity (η).



1. Young's modulus of elasticity

It is defined as the ratio of the normal stress to the longitudinal strain.

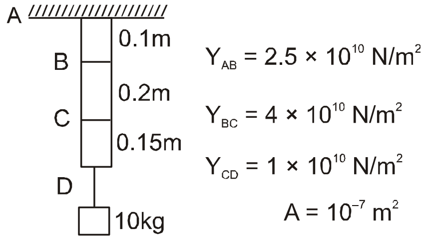
i.e. Young's modulus (Y) = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

Normal stress = F/A,

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

Example 3. Find out the shift in point B, C and D



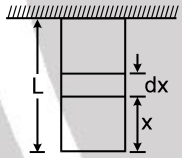
Solution : $\Delta L_B = \Delta L_{AB} = \dfrac{FL}{AY} = \dfrac{MgL}{AY} = \dfrac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3}$ m = 4mm

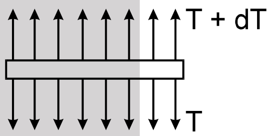
$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \dfrac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}} = 4 \times 10^{-3} + 5 \times 10^{-3} = 9$mm

$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \dfrac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}} = 9 \times 10^{-3} + 15 \times 10^{-3} = 24$ mm

ELONGATION OF ROD UNDER IT'S SELF WEIGHT

Let rod is having self weight 'W', area of cross-section 'A" and length 'L'. Considering on element at a distance 'x' from bottom.





then $T = \dfrac{W}{L}x$

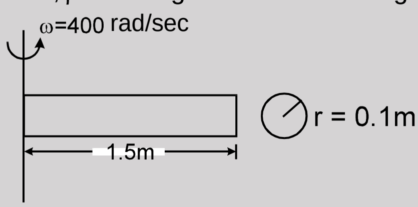
elongation in 'dx' element = $\dfrac{Tdx}{AY}$

Total elongation $s = \int_0^L \dfrac{Tdx}{AY} = \int_0^L \dfrac{Wx \ \ dx}{LAY} = \dfrac{WL}{2AY}$

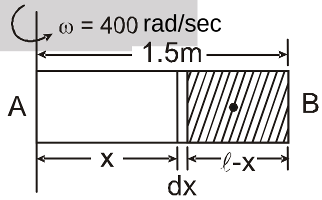
Note : One can do directly by considering total weight at C.M. and using effective length $\ell/2$.

Solved Example

Example 4. Given $Y = 2 \times 10^{11}$ N/m^2, $\rho = 10^4$ kg/m^3. Find out elongation in rod.



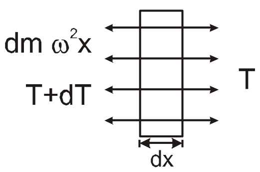
Solution : mass of shaded portion

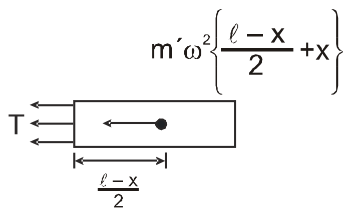


$m' = \dfrac{m}{\ell}(\ell - x)$ [where m = total mass = $\rho A \lambda$]

$T = m'\omega^2\left[\dfrac{\ell - x}{2} + x\right]$

$\Rightarrow \quad T = \dfrac{m}{\ell}(\ell - x)\omega^2\left(\dfrac{\ell + x}{2}\right) \qquad T = \dfrac{m\omega^2}{2\ell}(\ell^2 - x^2)$





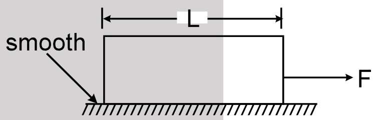
this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' = $\frac{Tdx}{AY}$

Total elongation $\delta = \int \frac{Tdx}{AY} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell AY} dx$

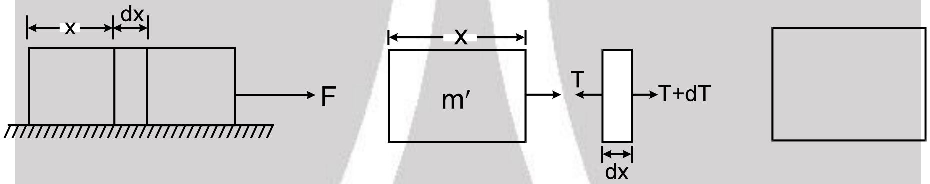
$$\delta = \frac{m\omega^2}{2\ell \quad AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

$$\delta = \frac{\rho\omega^2\ell^3}{3Y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}} = 9 \times 10^{-3} \text{ m} = 9\text{mm}$$

Example 5. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and Y respectively.



Solution :



Acceleration, $a = \frac{F}{m}$ then $T = m'a$ where $\Rightarrow$ $m' = \frac{m}{\ell}x$

$T = \frac{m}{\ell} x \frac{F}{m} = \frac{F \; x}{\ell}$

Elongation in element 'dx' = $\frac{Tdx}{AY}$

total elongation, $\delta = \int_0^{\ell} \frac{Tdx}{AY}$ $\qquad d = \int_0^{\ell} \frac{Fxdx}{A\ell Y} = \frac{F\ell}{2AY}$

Note : Try this problem, if friction is given between block and surface (μ = friction coefficient), and
Case : (I) $F < \mu mg$ (II) $F > \mu mg$

Ans. In both cases answer will be $\frac{F\ell}{2AY}$

2. Bulk modulus :

It is defined as the ratio of the normal stress to the volume strain

i.e. $B = \frac{\text{Pressure}}{\text{Volume strain}}$

The stress being the normal force applied per unit area and is equal to the pressure applied (p).

$$B = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in SI is $N^{-1} m^2$ or pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids, and for gases it is 10^{-8} times of solids.

$B_{solids} > B_{liquids} > B_{gases}$

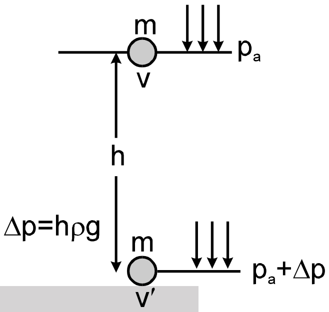
Isothermal bulk modulus of elasticity of gas B = P (pressure of gas)

Adiabatic bulk modulus of elasticity of gas $B = \gamma \times P$ where $\gamma = \frac{C_p}{C_v}$.

Solved Example

Example 6. Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility K = 50 × 10^{-6} /atm.

Solution : $B = \dfrac{\Delta p}{-\dfrac{\Delta V}{V}}$ $\quad \dfrac{\Delta V}{V} = -\dfrac{\Delta p}{B}$



m = ρV = const.

$d\rho\, V + dV\,.\,\rho = 0 \quad \Rightarrow \quad \dfrac{d\rho}{\rho} = -\dfrac{dV}{V}$

i.e. $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta p}{B} \quad \Rightarrow \quad \dfrac{\Delta\rho}{\rho} = \dfrac{1}{100}$

$\dfrac{1}{100} = \dfrac{h\rho g}{B}$ [assuming ρ = const.]

$h\,\rho\,g = \dfrac{B}{100} = \dfrac{1}{100K} \quad \Rightarrow \quad h\,\rho\,g = \dfrac{1\times10^5}{100\times50\times10^{-6}}$

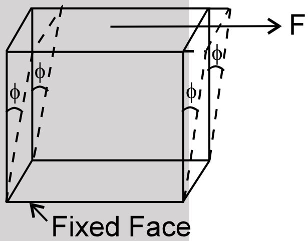
$h = \dfrac{10^5}{5000\times10^{-6}\times1000\times10} = \dfrac{100\times10^3}{50}$ = 2km **Ans.**

3. **Modulus of Rigidity :**

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.

∴ Tangential stress = F/A.

Let the vertical sides of the cube shifts through an angle θ, called shear strain



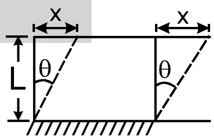
∴ Modulus of rigidity is given by

$\eta = \dfrac{\text{Tangential stress}}{\text{Shear strain}}$ or $\eta = \dfrac{F/A}{\phi} = \dfrac{F}{A\phi}$

Solved Example

Example 7. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4 × 10^6 N/m^2.

Solution : L = 5 × 10^{-2} m $\Rightarrow \dfrac{F}{A} = \eta\,\dfrac{x}{L}$



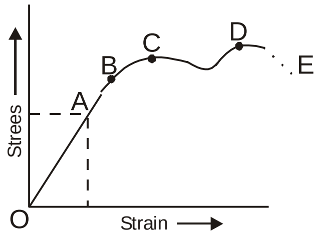
strain $\theta = \dfrac{F}{A\eta} = \dfrac{1800}{25\times10^{-4}\times2.4\times10^6} = \dfrac{180}{25\times24} = \dfrac{3}{10}$ = 0.3 radian

$\dfrac{x}{L}$ = 0.3 ⇒ x = 0.3 × 5 × 10^{-2} = 1.5 × 10^{-2} m = 1.5 cm **Ans.**

VARIATION OF STRAIN WITH STRESS

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. In shown figure, this type of behavior is represented by OB portion of the graph. Till

A the stress is proportional to strain and from A to B if deforming forces are removed then the wire comes to its original length but here stress is not proportional to strain.



OA → Limit of Proportionality
OB → Elastic limit
C → Yield Point
CD → Plastic behaviour
D → Ultimate point
DE → Fracture

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behavior is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

Important points

- Breaking stress = Breaking force/area of cross section.
- Breaking stress is constant for a material.
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that safety factor = breaking stress/working stress may have a large value.
- Breaking strain = elongation or compression/original dimension.
- Breaking strain is constant for a material.

Elastic after effect

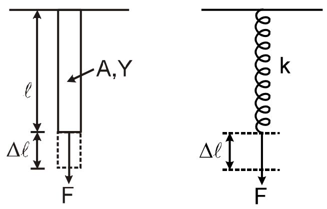
We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz fiber and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers.

For glass fiber elastic after effect is very large. It takes hours for glass fiber to return to its original state on removal of deforming force.

Elastic Fatigue

The, the loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a longtime of their use.

Analogy of Rod as a spring

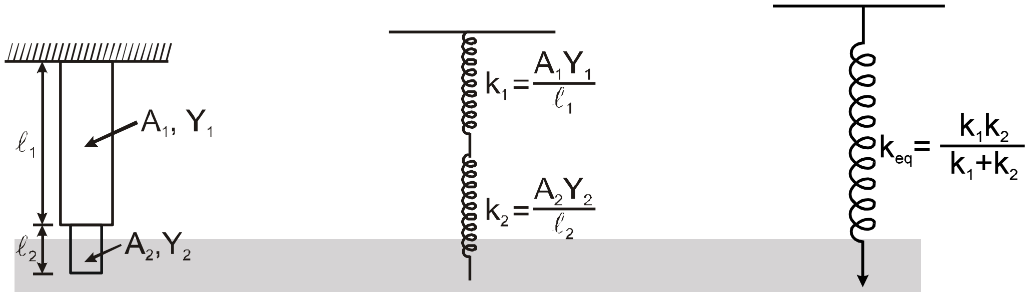


$$Y = \frac{\text{stress}}{\text{strain}} \quad \Rightarrow \quad Y = \frac{F\ell}{A\Delta\ell}$$

or $F = \frac{AY}{\ell}\Delta\ell$

$\frac{AY}{\ell}$ = constant, depends on type of material and geometry of rod. $F = k\Delta\ell$

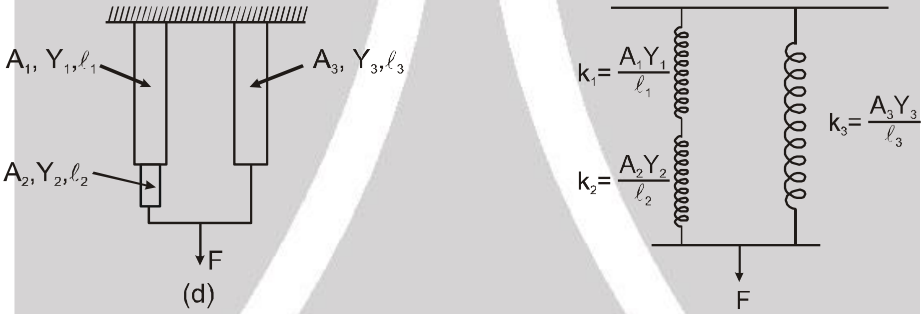
where $k = \frac{AY}{\ell}$ = equivalent spring constant.



(a) (b) (c)

for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.

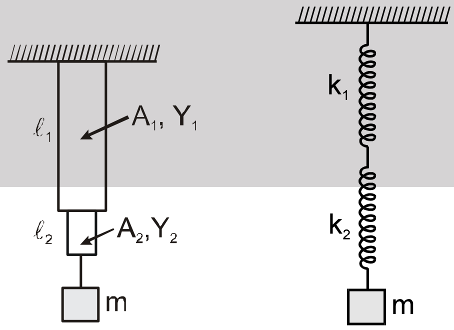
Figure (d) represents another combination of rods and their replaced spring system.



(d)

Solved Example

Example 8. A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.

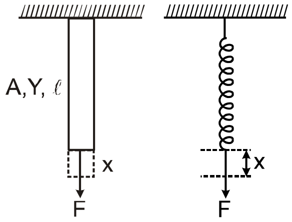


Solution : $k_{eq} = \frac{k_1k_2}{k_1 + k_2}$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$$

where $k_1 = \frac{A_1Y_1}{\ell_1}$ and $k_2 = \frac{A_2Y_2}{\ell_2}$

ELASTIC POTENTIAL ENERGY STORED IN A STRETCHED WIRE OR IN A ROD



Strain energy stored in equivalent spring

$$U = \frac{1}{2}kx^2$$

where $x = \frac{F\ell}{AY}$, $k = \frac{AY}{\ell}$

$$U = \frac{1}{2}\frac{AY}{\ell}\frac{F^2\ell^2}{A^2Y^2} = \frac{1}{2}\frac{F^2\ell}{AY}$$

equation can be re-arranged

$$U = \frac{1}{2}\frac{F^2}{A^2} \times \frac{\ell A}{Y} \quad [\ell A = \text{volume of rod, } F/A = \text{stress}]$$

$$U = \frac{1}{2}\frac{(\text{stress})^2}{Y} \times \text{volume}$$

again, $U = \frac{1}{2}\frac{F}{A} \times \frac{F}{AY} \times A\,\ell$ $\quad$ [Strain = $\frac{F}{AY}$]

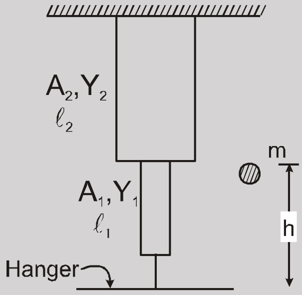
$$U = \frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}$$

again, $U = \frac{1}{2}\frac{F^2}{A^2Y^2}A\ell Y \Rightarrow$ $\quad U = \frac{1}{2}Y(\text{strain})^2 \times \text{volume}$

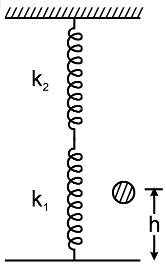
strain energy density $= \frac{\text{strain energy}}{\text{volume}} = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2}\text{stress} \times \text{strain}$

Solved Example

Example 9. A ball of mass 'm' drops from a height 'h', which sticks tomass-less hanger after striking. Neglect over turning, find out the maximum extension in rod. Assuming rod is massless.



Solution : Applying energy conservation $mg\,(h + x) = \frac{1}{2}\frac{k_1k_2}{k_1 + k_2}x^2$



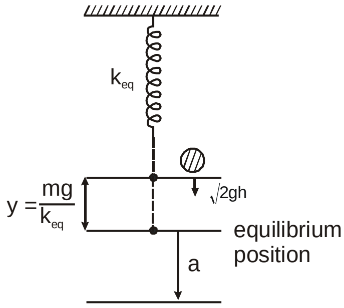
where $k_1 = \frac{A_1Y_1}{\ell_1}$ $\quad k_2 = \frac{A_2Y_2}{\ell_2}$

& $\quad k_{eq} = \frac{A_1A_2Y_1Y_2}{A_1Y_1\ell_2 + A_2Y_2\ell_1}$

$k_{eq}x^2 - 2mgx - 2mgh = 0$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk_{eq}}}{2k_{eq}}, \quad x_{max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$$

OTHERWAY BY S.H.M.



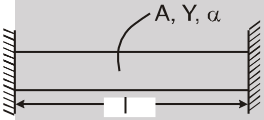
$\omega = \sqrt{\frac{k_{eq}}{m}}$ $v = \omega\sqrt{a^2 - y^2}$

$\sqrt{2gh} = \sqrt{\frac{k_{eq}}{m}}\sqrt{a^2 - y^2}$ $\Rightarrow$ $\sqrt{\frac{2mgh}{k_{eq}} + \frac{m^2g^2}{k_{eq}^2}} = a$

max^m extension

$= a + y = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}} + \frac{2mgh}{k_{eq}}}$

THERMAL STRESS :



If temp of rod is increased by ΔT, then change in length $\Delta\ell = \ell\ \alpha\ \Delta T$

strain $= \frac{\Delta\ell}{\ell} = \alpha\ \ \Delta T$

But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

$Y = \frac{\text{stress}}{\text{strain}}$

If ΔT = positive

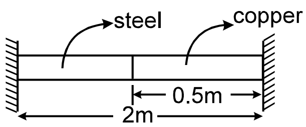
thermal stress = Y strain = $Y\ \alpha\ \Delta T$

If ΔT = negative

$\frac{F}{A} = Y\alpha\Delta T$ $F = AY\ \alpha\ \Delta T$

Solved Example

Example 10. When composite rod is free, then composite length increases to 2.002 m for temperature rise from 20ºC to 120ºC. When composite rod is fixed between the support, there is no change in component length find Y and α of steel, if $Y_{cu} = 1.5 \times 10^{13}$ N/m^2 $\alpha_{cu} = 1.6 \times 10^{-5}$/ºC.



Solution : $\Delta\ell = \ell_s\ \alpha_s\ \Delta T + \ell_c\ \alpha_c\Delta T$

$.002 = [1.5\ \alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$

$\alpha_s = \frac{1.2\times10^{-5}}{1.5} = 8 \times 10^{-6}$/ºC there is no change in component length

For steel

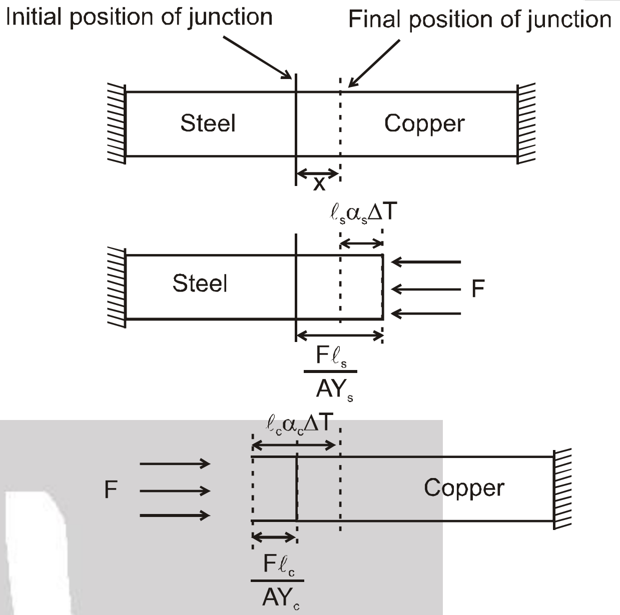
$x = \ell_s \alpha_s \Delta T - \frac{F\ell_s}{AY_s} = 0$

$\frac{F}{AY_s} = \alpha_s \Delta T$ ....(A)

for copper

$x = \frac{F\ell_c}{AY_c} - \ell_c \alpha_c \Delta T = 0$

$\frac{F}{AY_c} = \alpha_c \Delta T$ ...(B)



B/A $\Rightarrow \frac{Y_s}{Y_c} = \frac{\alpha_c}{\alpha_s}$

$Y_s = Y_c \frac{\alpha_c}{\alpha_s} = \frac{1.5\times10^{13}\times16\times10^{6}}{8\times10^{-6}}$

$= 3\times10^{13}$ N/m^2

APPLICATIONS OF ELASTICITY

Some of the important applications of the elasticity of the materials are discussed as follows :

1. The material used in bridges lose its elastic strength with time bridges are declared unsafe after long use.
2. **To estimate the maximum height of a mountain :**
   The pressure at the base of the mountain = hρg = stress. The elastic limit of a typical rock is 3×10^8 N m^{-2}
   The stress must be less than the elastic limits, otherwise the rock begins to flow.
   $h < \frac{3\times10^8}{\rho g} \Rightarrow h < 10^4$ m ($\because \rho = 3 \times 10^3$ kg m^{-3} ; g = 10 ms^{-2}) or h = 10 km
   It may be noted that the height of Mount Everest is nearly 9 km.

TORSION CONSTANT OF A WIRE

$C = \frac{\pi\eta r^4}{2\ell}$ Where η is modulus of rigidity r and ℓ is radius and length of wire respectively.

(a) Toque required for twisting by an angle θ, $\tau = C\theta$.

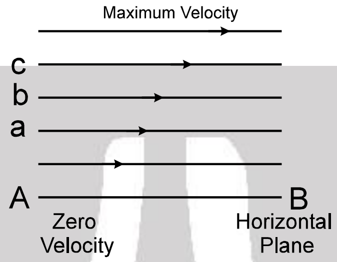
(b) Work done in twisting by an angle θ, $W = \frac{1}{2} C\theta^2$.

VISCOSITY

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c. Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**

The property of viscosity is seen in the following examples :



(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerin, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

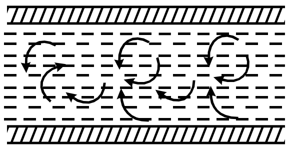
(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

(iv) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

FLOW OF LIQUID IN A TUBE: CRITICAL VELOCITY

When a liquid flows in a tube, the viscous forces oppose the flow of the liquid, Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag and eddy-currents are developed in it.



Reynold's proved that the critical velocity for a liquid flowing in a tube is $v_c = k\eta/\rho r$. where ρ is density and η is viscosity of the liquid, r is radius of the tube and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity, the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedily inspite of being very thick (of large viscosity).

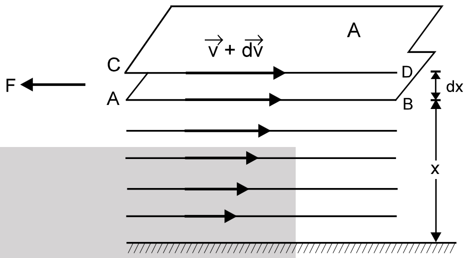
VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

The property of a liquid by virtue of which an opposing force (internal friction) comes into play when ever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + d\vec{v}$ at a distance x and (x + dx) respectively from the fixed solid surface.

According to Newton, the viscous drag or back ward force (F) between these layers depends.

(i) directly proportional to the area (A) of the layer and

(ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.



i.e. $F \propto A\frac{dv}{dx}$ or $F = -\eta A\frac{dv}{dx}$ ...(1)

η is called Coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.

2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.

3. Both are due to molecular attractions.

Differences between them →

Viscosity	Solid Friction
(i) Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i) Friction between two solids is independent of the area of solid surfaces in contact.
(ii) Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii) Friction is independent of the relative velocity between two surfaces.
(iii) Viscous drag is independent of normal reaction between two layers of liquid.	(iii) Friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.

2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.

4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.

5. It finds an important use in the circulation of blood through arteries and veins of human body.

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have $\eta = \dfrac{F}{A(\Delta v_x / \Delta z)}$

$\therefore$ dimensions of $\eta = \dfrac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \dfrac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$

Its unit is kg/(meter-second)*

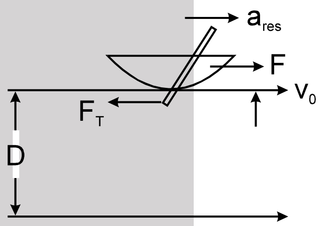
In C.G.S. system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N sm^{-2} and is called decapoise.

1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

Solved Example

Example 11. A man is rowing a boat with a constant velocity 'v_0' in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.

Solution : $F - F_T = m\, a_{res}$



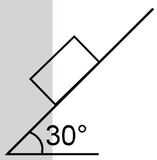
As boat moves with constant velocity $a_{res} = 0$

$F = F_T$

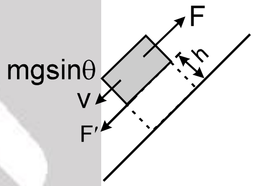
But $F_T = \eta A \dfrac{dv}{dz}$, but $\dfrac{dv}{dz} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$

then $F = F_T = \dfrac{\eta A v_0}{D}$

Example 12. A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. (g = 10 m/sec^2) find out the thickness of layer of liquid.



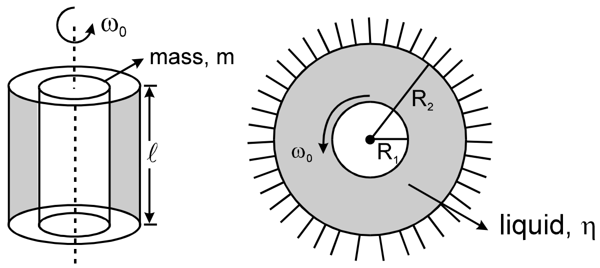
Solution : $F = \eta A \dfrac{dv}{dz} = mg \sin\theta$ $\quad \dfrac{dv}{dz} = \dfrac{v}{h}$



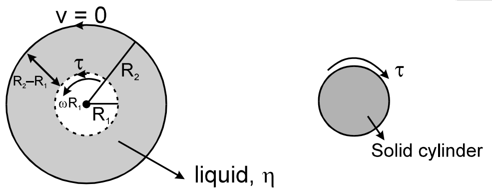
$20 \times 10 \times \sin 30° = \eta \times 4 \times \dfrac{10}{h}$

$h = \dfrac{40 \times 10^{-2}}{100}$ [$\eta = 10^{-1}$ poise $= 10^{-2}$ N-sec-m^{-2}] $= 4 \times 10^{-3}$ m = 4 mm

Example 13. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.



Solution : $F = \eta A \frac{dv}{dz}$, where $\frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$



$$F = \eta \frac{2\pi \quad R_1 \quad \ell \quad \omega \quad R_1}{R_2 - R_1}$$

and $\tau = FR_1 = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$$I\alpha = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

$$\Rightarrow \frac{mR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1} \quad \text{or} \quad -\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \int_0^t dt$$

$$\Rightarrow t = \frac{m(R_2 - R_1)\ell n2}{4\pi\eta\ell R_1}$$

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$ On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$

STOKE'S LAW

Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity η is given by $F = 6 \pi \eta r v$. This is called Stokes' law.

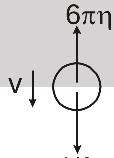
TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward = $V(\rho - \sigma) g = \frac{4}{3}\pi r^3 (\rho - \sigma)g$,



(ii) viscous force acting upward = $6 \pi \eta rv$.
Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi rv = p\, r^3 (\rho - \sigma) g \qquad \text{or} \quad v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Important point

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Solved Example

Example 14. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Solution : Rate of heat loss = power = F × v = $6\pi\eta r v \times v = 6\pi\eta r v^2 = 6p\,\eta\, r\left[\frac{2}{9}\frac{gr^2(\rho_0-\rho_\ell)}{\eta}\right]^2$

Rate of heat loss $\alpha\ r^5$

Example 15. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg /(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Solution : By Stoke's law , the terminal velocity of a water drop of radius r is given by $\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values :

$$\nu = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2\times(1.0\times10^3)\times9.8}{1.8\times10^{-5}} = \mathbf{2.72 \times 10^{-4}\ m/s}$$

Example 16. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given : coefficient of viscosity of water = 1.0×10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0×10^3 kg/m^3.

Solution : The velocity attained by the sphere in falling freely from a height h is

$\nu = \sqrt{2gh}$ ....(i)

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere σ (= 1.0×10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore\quad \nu = \frac{2\times(1.0\times10^{-3})^2(1.0\times10^4-1.0\times10^3)\times10}{9\times1.0\times10^{-3}} = 20\ \text{m/s}$$

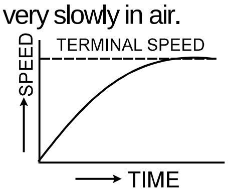
from equation (i), we have $h = \frac{\nu^2}{2g} = \frac{20\times20}{2\times10} = 20$ m

Applications of Stokes' Formula

(i) In determining the Electronic Charge by Millikan's Experiment : Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops : Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity,

(iii) Parachute : When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air. In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. In Fig graph is shown between the speed of the falling soldier and time.



Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

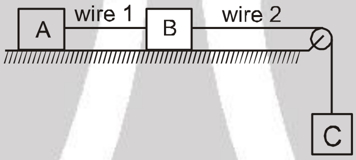
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Elastic behaviour, longitudinal stress, young modulus

A-1.✍ If a compressive force of 3.0×10^4 N is exerted on the end of 20 cm long bone of cross-sectional area 3.6cm^2,
(a) will the bone break and (b) if not, how much will it shorten?
Given, compressive strength of bone = 7.7×10^8 Nm^{-2} and Young's modulus of bone = 1.5×10^{10}Nm^{-2}

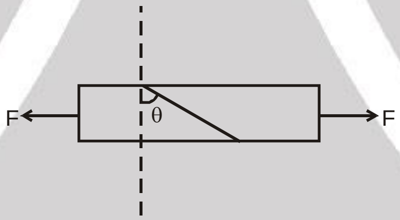
A-2. Two exactly similar wires of steel and copper are stretched by equal force. If the difference in their elongation is 0.5 cm, find by how much each wire has elongated. (Given Young's modulus for steel = 2×10^{12} dyne cm^{-2} and for copper 12×10^{11} dyne cm^{-2})

A-3.✍ Three blocks A, B and C each of mass 4 kg are attached as shown in figure. Both the wires has equal cross sectional area 5×10^{-7} m^2. The surface is smooth. Find the longitudinal strain in each wire if Young modulus of both the wires is 2×10^{11} N/m^2 (Take g = 10 m/s^2)



Section (B) : Tangential stress and strain, shear modulus

B-1.✍ A bar of cross-section A is subjected to equal and opposite tensile forces F at its ends. Consider a plane through the bar making an angle θ with a plane at right angles to the bar



(a) What is the tensile stress at this plane in terms of F, A and θ ?
(b) What is the shearing stress at the plane, in terms of F, A and θ ?
(c) For what value of θ is the tensile stress a maximum ?
(d) For what value of θ is the shearing stress a maximum?

Section (C) : Pressure and volumetric strain, bulk modulus of elasticity

C-1.✍ A spherical ball contracts in volume by 0.001% when it is subjected to a pressure of 100 atmosphere. Calculate its bulk modulus.

Section (D) : Elastic potential energy

D-1. Calculate the increase in energy of a brass bar of length 0.2 m and cross-sectional area 1 cm^2 when compressed with a load of 5 kg-weight along its length. (Young's modulus of brass = 1.0×10^{11} N/m^2 and g = 9.8 m/s^2).

D-2.✍ When the load on a wire increased slowly from 2 kg wt. to 4 kg wt., the elongation increases from 0.6 mm to 1.00 mm. How much work is done during the extension of the wire? [g = 9.8 m/s^2]

Section (E) : Viscosity

E-1.✍ A spherical ball of radius 3.0×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. Viscosity of water is 9.8×10^{-6} N-s/m^2. [g = 9.8 m/s^2]

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Elastic behavior longitudinal stress, young modulus

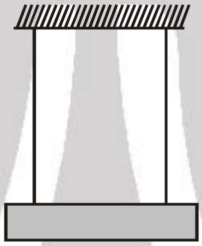
A-1. The diameter of a brass rod is 4 mm and Young's modulus of brass is 9×10^{10} N/m^2. The force required to stretch it by 0.1% of its length is :

(A) $360\,\pi$ N (B) 36 N (C) $144\,\pi \times 10^3$ N (D) $36\,\pi \times 10^5$ N

A-2. A steel wire is suspended vertically from a rigid support. When loaded with a weight in air, it expands by L_a and when the weight is immersed completely in water, the extension is reduced to L_w. Then relative density of the material of the weight is

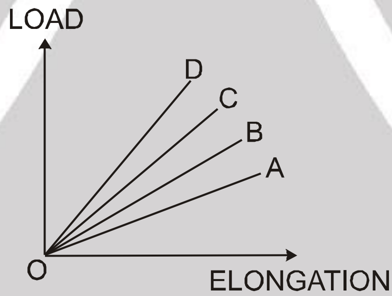
(A) $\frac{L_a}{L_a - L_w}$ (B) $\frac{L_w}{L_a}$ (C) $\frac{L_a}{L_w}$ (D) $\frac{L_w}{L_a - L_w}$

A-3. Two wires of equal length and cross-section area suspended as shown in figure. Their Young's modulus are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



(A) $Y_1 + Y_2$ (B) $\frac{Y_1 + Y_2}{2}$ (C) $\frac{Y_1 Y_2}{Y_1 + Y_2}$ (D) $\sqrt{Y_1 Y_2}$

A-4. The load versus elongation graph for four wires of the same materials is shown in the figure. The thinnest wire is represented by the line :



(A) OC (B) OD (C) OA (D) OB

Section (B) : Tangential stress and strain, shear modulus

B-1. A square brass plate of side 1.0 m and thickness 0.005 m is subjected to a force F on its smaller opposite edges, causing a displacement of 0.02 cm. If the shear modulus of brass is 0.4×10^{11} N/m^2, the value of the force F is

(A) 4×10^3 N (B) 400 N (C) 4×10^4 N (D) 1000 N

Section (C) : Pressure and volumetric strain, bulk modulus of elasticity

C-1. A metal block is experiencing an atmospheric pressure of 1×10^5 N/m^2, when the same block is placed in a vacuum chamber, the fractional change in its volume is (the bulk modulus of metal is 1.25×10^{11} N/m^2)

(A) 4×10^{-7} (B) 2×10^{-7} (C) 8×10^{-7} (D) 1×10^{-7}

Section (D) : Elastic potential energy

D-1. If the potential energy of a spring is V on stretching it by 2 cm, then its potential energy when it is stretched by 10 cm will be :

(A) V/25 (B) 5 V (C) V/5 (D) 25 V

D-2. If work done in stretching a wire by 1mm is 2J, the work necessary for stretching another wire of same material, but with double the radius and half the length by 1mm in joule is –
(A) 1/4 (B) 4 (C) 8 (D) 16

Section (E) : Viscosity

E-1. An oil drop falls through air with a terminal velocity of 5×10^{-4} m/s.
(i) the radius of the drop will be :
(A) 2.5×10^{-6} m (B) 2×10^{-6} m (C) 3×10^{-6} m (D) 4×10^{-6} m

(ii) the terminal velocity of a drop of half of this radius will be : (Viscosity of air $= \frac{18 \times 10^{-5}}{5}$ N-s/m^2, g = 10 m/s^2, density of oil = 900 Kg/m^3. Neglect density of air as compared to that of oil)
(A) 3.25×10^{-4} m/s (B) 2.10×10^{-4} m/s (C) 1.5×10^{-4} m/s (D) 1.25×10^{-4} m/s

E-2. The terminal velocity of a sphere moving through a viscous medium is :
(A) directly proportional to the radius of the sphere
(B) inversely proportional to the radius of the sphere
(C) directly proportional to the square of the radius of sphere
(D) inversely proportional to the square of the radius of sphere

E-3. A sphere is dropped gently into a medium of infinite extent. As the sphere falls, the force acting downwards on it
(A) remains constant throughout
(B) increases for sometime and then becomes constant
(C) decreases for sometime and then becomes zero
(D) increases for sometime and then decreases.

E-4. A solid sphere falls with a terminal velocity of 10 m/s in air. If it is allowed to fall in vacuum,
(A) terminal velocity will be more than 10 m/s (B) terminal velocity will be less than 10 m/s
(C) terminal velocity will be 10 m/s (D) there will be no terminal velocity

PART - III : MATCH THE COLUMN

1. A metal wire of length L is suspended vertically from a rigid support. When a bob of mass M is attached to the lower end of wire, the elongation of the wire is ℓ :

Column - I	Column - II
(A) The loss in gravitational potential energy of mass M is equal to	(p) $Mg\ell$
(B) The elastic potential energy stored in the wire is equal to	(q) $\frac{1}{2}Mg\ell$
(C) The elastic constant of the wire is equal to	(r) Mg/ℓ
(D) Heat produced during extension is equal to	(s) $\frac{1}{4}Mg\ell$

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ A force F is needed to break a copper wire having radius R. The force needed to break a copper wire of radius 2 R will be :

(A) F/2 (B) 2 F (C) 4 F (D) F/4

2. Two hail stones with radii in the ratio of 1 : 2 fall from a great height through the atmosphere. Then the ratio of their momentum after they have attained terminal velocity is

(A) 1 : 1 (B) 1 : 4 (C) 1 : 16 (D) 1 : 32

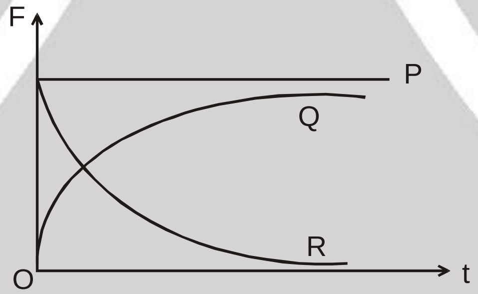
3.✍ A 50 kg motor rests on four cylindrical rubber blocks. Each block has a height of 4 cm and a cross-sectional area of 16 cm^2. The shear modulus of rubber is 2×10^6 N/m^2. A sideways force of 500 N is applied to the motor. The distance that the motor moves sideways is

(A) 0.156 cm (B) 1.56 cm (C) 0.312 cm (D) 0.204 cm

4.✍ A brass rod of length 2 m and cross-sectional area 2.0 cm^2 is attached end to end to a steel rod of length L and cross-sectional area 1.0 cm^2. The compound rod is subjected to equal and opposite pulls of magnitude 5×10^4 N at its ends. If the elongations of the two rods are equal, the length of the steel rod (L) is ($Y_{Brass} = 1.0 \times 10^{11}$ N/m^2 and $Y_{Steel} = 2.0 \times 10^{11}$ N/m^2)

(A) 1.5 m (B) 1.8 m (C) 1 m (D) 2 m

5. A spherical ball is dropped in a long column of viscous liquid. Which of the following graphs represent the variation of



(i) gravitational force with time

(ii) viscous force with time

(iii) net force acting on the ball with time

(A) Q, R, P (B) R, Q, P (C) P, Q, R (D) R, P, Q

6. The compressibility of water is 46.4×10^{-6}/atm. This means that

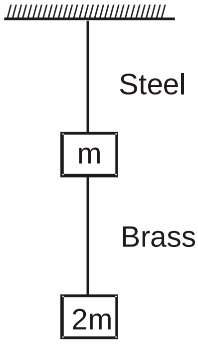
(A) the bulk modulus of water is 46.4×10^6 atm

(B) volume of water decreases by 46.4 one-millionths of the original volume for each atmosphere increase in pressure

(C) when water is subjected to an additional pressure of one atmosphere, its volume decreases by 46.4%

(D) When water is subjected to an additional pressure of one atmosphere, its volume is reduced to 10^{-6} of its original volume.

7. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are a, b and c respectively. Then the corresponding ratio of increase in their lengths would be :



(A) $\frac{2ac}{b^2}$ (B) $\frac{3a}{2b^2c}$ (C) $\frac{3c}{2ab^2}$ (D) $\frac{2a^2c}{b}$

8. If a rubber ball is taken at the depth of 200 m in a pool its volume decreases by 0.1%. If the density of the water is 1×10^3 kg/m^3 and g = 10 m/s^2, then the volume elasticity in N/m^2 will be :
(A) 10^8 (B) 2×10^8 (C) 10^9 (D) 2×10^9

9. Two wires of the same material and length but diameter in the ratio 1 : 2 are stretched by the same force. The ratio of potential energy per unit volume for the two wires when stretched will be :
(A) 1 : 1 (B) 2 : 1 (C) 4 : 1 (D) 16 : 1

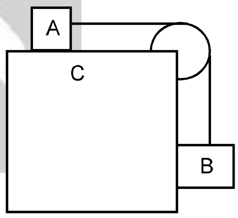
10. A small steel ball falls through a syrup at constant speed of 10 cm/s. If the steel ball is pulled upwards with a force equal to twice its effective weight, how fast will it move upwards ?
(A) 10 cm/s (B) 20 cm/s (C) 5 cm/s (D) – 5 cm/s

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A rod 1 m long is 10 cm^2 in area for a portion of its length and 5 cm^2 in area for the remaining. The strain energy of this stepped bar is 40 % of that a bar 10 cm^2 in area and 1 m long under the same maximum stress. What is the length of the portion 10 cm^2 in area.

2. The cross-section of a bar is given by $\left[1+\frac{x^2}{100}\right]$ cm^2, where 'x' is the distance from one end. If the extension under a load of '20 kN' on a length of 10 cm is $\lambda \times 10^{-3}$ cm then find λ. Y = 2×10^5 N/mm^2.

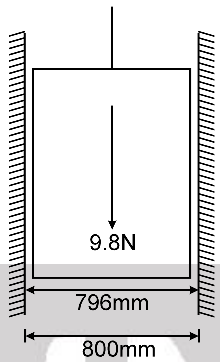
3. Two block A and B are connected to each other by a string, passing over a frictionless pulley as shown in figure. Block A slides over the horizontal top surface of a stationary block C and block B slide along the vertical side of C both with uniform speed. The coefficient of friction between the surface of blocks is 0.2. String stiffness is 2000 N/m. If mass of block B is 2 kg. Calculate ratio (in kg/J) of the mass of block A and the energy stored in the string.



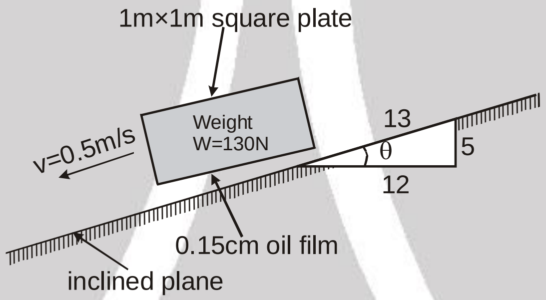
4. A thin ring of radius R is made of a material of density ρ and Young's modulus Y. If the ring is rotated about its centre in its own plane with angular velocity ω, if the small increase in its radius is $\frac{2\rho\omega^2R^3}{\lambda Y}$ then find λ.

5. A uniform copper bar of density ρ, length L, cross-sectional area S and Young's modulus Y is moving on a frictionless horizontal surface with constant acceleration a_0. If total elongation in the wire is $\frac{\rho a_0 L^2}{\lambda Y}$ then find λ.

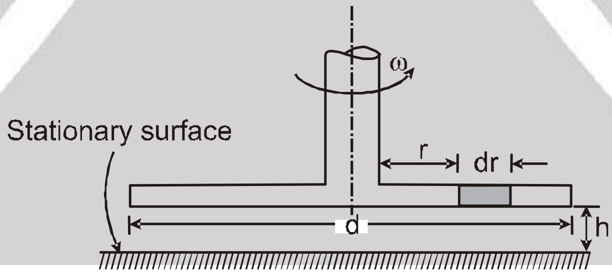
6. A piston of 796 mm diameter and 200 mm long works in a cylinder of 800 mm diameter as shown in figure. If the annular space is filled with a lubricating oil of viscosity 5 centipoises, calculate the constant speed (nearest to integer) (in m/s) of descent of piston in vertical position. The weight of piston and the axial load are 9.8 N.



7. If the approximate viscosity of the oil for the following case is $\lambda \times 10^{-2}$ Ns/m^2 then find λ :

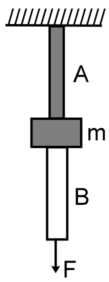


8. A circular disc of a diameter 'd' is slowly rotated in a liquid of large viscosity 'η' at a small distance 'h' from a fixed surface as shown in figure. If an expression for torque 'τ' necessary to maintain an angular velocity 'ω' is $\frac{\pi\eta\omega d^4}{\lambda h}$ then find λ.



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The wires A and B shown in the figure, are made of the same material and have radii r_A and r_B. A block of mass m kg is tied between them : If the force F is mg/3, one of the wires breaks.



(A) A will break before B if $r_A < 2r_B$
(B) A will break before B if $r_A = r_B$
(C) Either A or B will break if $r_A = 2r_B$
(D) The lengths of A and B must be known to decide which wire will break

2. A small ball bearing is released at the top of a long vertical column of glycerine of height 2h. The bal bearing falls through a height h in a time t_1 and then the remaining height with the terminal velocity in time t_2. Let W_1 and W_2 be the work done against viscous drag over these height. Therefore,

(A) $t_1 < t_2$ (B) $t_1 > t_2$ (C) $W_1 = W_2$ (D) $W_1 < W_2$

3.✍ A metal wire of length L area of cross-section A and Young's modulus Y is stretched by a variable force F such that F is always slightly greater than the elastic force of resistance in the wire. When the elongation of the wire is ℓ :

(A) the work done by F is $\frac{YA^2}{L}$

(B) the work done by F is $\frac{YA\ell^2}{2L}$

(C) the elastic potential energy stored in the wire is $\frac{YA\ell^2}{2L}$

(D) heat is produced during the elongation

PART - IV : COMPREHENSION

Comprehension-1

When a tensile or compressive load 'P' is applied to rod or cable, its length changes. The change in length x which, for an elastic material is proportional to the force (Hook's law).

$P \propto x$ or $P = kx$

The above equation is similar to the equation of spring. For a rod of length L, area A and young modulus Y, the extension x can be expressed as -

$x = \frac{PL}{AY}$ or $P = \frac{AY}{L}x$, hence $K = \frac{AY}{L}$

Thus rods or cables attached to lift can be treated as springs. The energy stored in rod is called strain energy & equal to $\frac{1}{2}$ Px. The loads placed or dropped on the floor of lift cause stresses in the cables and can be evaluated by spring analogy. If the cable of lift is previously stressed and load is placed or dropped, then maximum extension in cable can be calculated by energy conservation.

1. If rod of length 4 m, area 4cm^2 and young modulus 2×10^{10} N/m^2 is attached with mass 200 kg, then angular frequency of SHM (rad/sec.) of mass is equal to -

(A) 1000 (B) 10 (C) 100 (D) 10π

2. In above problem if mass of 10 kg falls on the massless collar attached to rod from the height of 99cm then maximum extension in the rod is equal (g = 10 m/sec^2) -

(A) 9.9 cm (B) 10 cm (C) 0.99 cm (D) 1 cm

3.✍ In the above problem, the maximum stress developed in the rod is equal to - (N/m^2)

(A) 5×10^7 (B) 5×10^8 (C) 4×10^7 (D) 4×10^8

4.✍ If two rods of same length (4m) and cross section areas 2 cm^2 and 4 cm^2 with same young modulus 2×10^{10} N/m^2 are attached one after the other with mass 600 kg then angular frequency is -

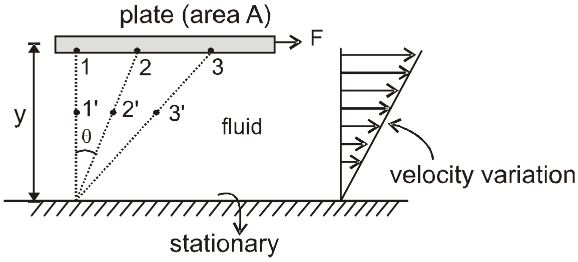
(A) $\frac{1000}{3}$ (B) $\frac{10}{3}$ (C) $\frac{100}{3}$ (D) $\frac{10\pi}{3}$

5.✍ Four identical rods of geometry as described in problem (2) are attached with lift. If weight of the lift cage is 1000 N, and elastic limit of each rod is taken as 9×10^6N/m^2 then the number of persons it can carry safely is equal to. (g = 10m/sec^2, assume average mass of a person as 50 kg and lift moves with constant speed)

(A) 7 (B) 26 (C) 24 (D) 25

Comprehension-2

Viscosity is the property of fluid by virtue of which fluid offers resistance to deformation under the influence of a tangential force.

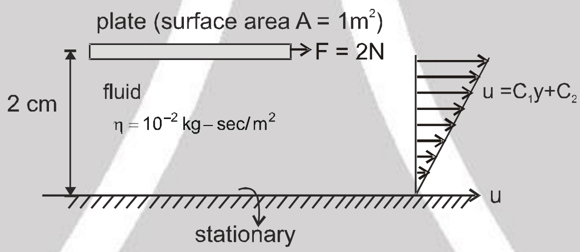


In the given figure as the plate moves the fluid particle at the surface moves from position 1 to 2 and so on, but particles at the bottom boundary remains stationary. If the gap between plate and bottom boundary is small, fluid particles in between plate and bottom moves with velocities as shown by linear velocity distribution curve otherwise the velocity distribution may be parabolic. As per Newton's law of viscosity the tangential force is related to time rate of deformation -

$\frac{F}{A} \alpha \frac{d\theta}{dt}$ but $y\frac{d\theta}{dt} = u$, $\frac{d\theta}{dt} = \frac{u}{y}$ then $F = \eta A \frac{u}{y}$, η = coefficient of viscosity

for non-linear velocity distribution $F = \eta A \frac{du}{dy}$ where $\frac{u}{y}$ or $\frac{du}{dy}$ is known as velocity gradient.

6. In the given figure if force of 2N is required to maintain constant velocity of plate, the value of constant C_1 & C_2 are -

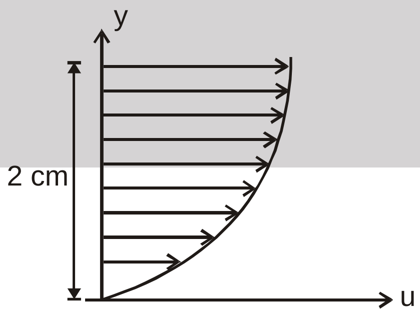


(A) 100, 100 (B) 0, 100 (C) 200, 0 (D) 0, 200

7. In previous question the value of constant speed of plate (m/sec.) is equal to -

(A) 0 (B) 4 (C) 2 (D) 1

8. If velocity distribution is given as (parabolic) $u = c_1y^2 + c_2y + c_3$



for the same force of 2N and the speed of the plate 2 m/sec, the constants C_1, C_2 & C_3 are-

(A) 200, 200, 0 (B) 5000, 200, 0 (C) 5000, 0, 0 (D) 500, 200, 0

9. The velocity gradient just below the plate. in above problem is equal to - (per second)

(A) Zero (B) 100 (C) 500 (D) 200

10. The velocity gradient just near the bottom boundary is equal to -

(A) Zero (B) 100 (C) 500 (iv) 200

Exercise-3

✍ Marked Questions can be used as for Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1.✍ A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity. **[JEE 2004, 2/60]**

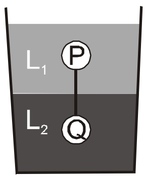
2.✍ A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is : **[JEE 2010, 3/252]**

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m s^{-1}. Given g = 9.8 ms^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is : **[JEE 2010, 5/237, –2]**
(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C (C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

4.✍ One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by a applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is :
[JEE (Advanced) 2013, 3/60,–1]
(A) 0.25 (B) 0.50 (C) 2.00 (D) 4.00

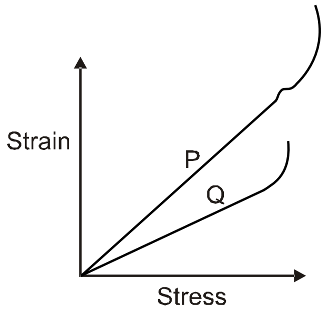
5. Durring Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **[JEE (Advanced) 2014, P-1, 3/60]**

6*. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then **[JEE (Advanced) 2015 ; P-2,4/88, –2]**



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$ (C) $\vec{V}_P.\vec{V}_Q > 0$ (D) $\vec{V}_P.\vec{V}_Q < 0$

7*. In plotting stress versus strain curves for the materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



[JEE (Advanced) 2015 ; P-2,4/88, –2]

(A) P has more tensile strength than Q
(B) P is more ductile than Q
(C) P is more brittle than Q
(D) The Young's modulus of P is more than that of Q.

8. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is : **[JEE (Advanced) 2016 ; P-1, 3/62]**

9*. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true? **[JEE (Advanced) 2018, P-2, 4/60, –2]**
(A) The resistive force of liquid on the plate is inversely proportional to h
(B) The resistive force of liquid on the plate is independent of the area of the plate
(C) The tangential (shear) stress on the floor of the tank increases with u_0
(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

10. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and τ = 4s. The displacement of the block, in metres, at t = τ is __________. Take e^{-1} = 0.37. **[JEE (Advanced) 2018, P-2, 3/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity ν. The retarding viscous force acting on the spherical ball is : **[AIEEE 2004, 3/225, –1]**

(1) directly proportional to R but inversely proportional to ν

(2) directly proportional to both radius R and velocity ν

(3) inversely proportional to both radius R and velocity ν

(4) inversely proportional to R but directly proportional to ν

2. If 'S' is stress and 'Y' is Young's modulus of material of a wire, the energy stored in the wire per unit volume is : **[AIEEE-2005, 3/225, –1]**

(1) $2S^2Y$ (2) $\frac{S^2}{2Y}$ (3) $\frac{2Y}{S^2}$ (4) $\frac{S}{2Y}$

3. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid then find the terminal speed of sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid (density = 1.5 kg/m^3). **[AIEEE 2006, 3/165, –1]**
(1) 0.4 m/s (2) 0.133 m/s (3) 0.1 m/s (4) 0.2 m/s

4. A wire elongates by ℓ mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm) **[AIEEE 2006, 3/165, –1]**

(1) ℓ (2) 2ℓ (3) zero (4) $\ell/2$

5. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is **[AIEEE-2008, 3/105]**

(1) $\frac{Vg\rho_1}{k}$ (2) $\sqrt{\frac{Vg\rho_1}{k}}$ (3) $\frac{Vg(\rho_1-\rho_2)}{k}$ (4) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$

6. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[AIEEE-2009, 4/144]**

(1) 4F (2) 6F (3) 9F (4) F

7. If a ball of steel (density $p = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in a water (Coefficient of Viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($p = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s.) would be, nearly : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) 6.25×10^{-4} cms^{-1} (2) 6.45×10^{-4}cms^{-1} (3) 1.5×10^{-5} cms^{-1} (4) 1.6×10^{-5} cms^{-1}

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is : (For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1}) **[JEE (Main) 2014 ; 4/120, –1]**

(1) 2.2×10^8 Pa (2) 2.2×10^9 Pa (3) 2.2×10^7 Pa (4) 2.2×10^6 Pa

9. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to : (g = gravitational acceleration) **[JEE (Main) 2015; 4/120, –1]**

(1) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$ (3) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

10. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire crosssection of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is : **[JEE (Main) 2018; 4/120, –1]**

(1) $\frac{mg}{3Ka}$ (2) $\frac{mg}{Ka}$ (3) $\frac{Ka}{mg}$ (4) $\frac{Ka}{3mg}$

Answers

EXERCISE-1

PART – I

Section (A)

A-1. No, $\frac{10}{9}$ x 10^{-3} m = 1.11 mm.

A-2. 0.75 cm, 1.25 cm

A-3. $\frac{4}{3}\times10^{-4}$, $\frac{8}{3}\times10^{-4}$

Section (B) :

B-1. (a) $\frac{F\cos^2\theta}{A}$ (b) $\frac{F\sin2\theta}{2A}$

(c) $\theta = 0^o$ (d) $\theta = 45^o$

Section (C) :

C-1. 10^7 atmosphere

Section (D) :

D-1. 2.4×10^{-5} J. **D-2.** 13.72×10^{-3} J

Section (E) :

E-1. $\frac{81}{49}\times10^3$ m

PART - II

Section (A) :

A-1. (A) **A-2.** (A) **A-3.** (B)
A-4. (C)

Section (B) :

B-1. (C)

Section (C) :

C-1. (C)

Section (D) :

D-1. (D) **D-2.** (D)

Section (E) :

E-1. (i) (C) ; (ii) (D) **E-2.** (C)
E-3. (C) **E-4.** (D)

PART - III

1. (A) $\rightarrow$ p ; (B) $\rightarrow$ q ; (C) $\rightarrow$ r ; (D) $\rightarrow$ q

EXERCISE-2

PART - I

1.	(C)	**2.**	(D)	**3.**	(A)
4.	(D)	**5.**	(C)	**6.**	(B)
7.	(C)	**8.**	(D)	**9.**	(D)
10.	(A)				

PART - II

1.	40	**2.**	8	**3.**	100
4.	2	**5.**	2	**6.**	8
7.	15	**8.**	32		

PART - III

1.	(ABC)	**2.**	(BD)	**3.**	(BC)

PART – IV

1.	(C)	**2.**	(D)	**3.**	(A)
4.	(C)	**5.**	(B)	**6.**	(C)
7.	(B)	**8.**	(C)	**9.**	(D)
10.	(A)				

EXERCISE-3

PART - I

1.	$\frac{dQ}{dt}\propto r^5$			**2.**	4
3.	(D)	**4.**	(C)	**5.**	8
6.	(AD)	**7.**	(AB)	**8.**	3
9.	(ACD)	**10.**	6.30		

PART - II

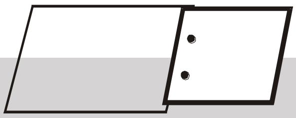
1.	(2)	**2.**	(2)	**3.**	(3)
4.	(1)	**5.**	(4)	**6.**	(3)
7.	(1)	**8.**	(1)	**9.**	(1)
10.	(1)				

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A wire loaded by a weight of density 7.6 g cm^{-3} is found to measure 90 cm. On immersing the weight in water, the length decreases by 0.18 cm. Find the original length of wire.

2. Two long metallic strips are joined together by two rivet each of radius 2.0 mm (see figure). Each rivet can withstand a maximum shearing stress of 1.5×10^9 Pa. What is the maximum tensile force that the strip can exert, assuming each rivet shares the stretching load equally?



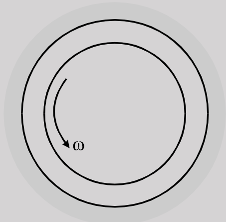
3. Eight rain drops of radius one mm each falling down with a terminal velocity of 5 cm s^{-1} coalesces to form a bigger drop. Calculate the terminal velocity of the bigger drop.

4. An air bubble of radius 1 cm is rising at a steady rate of 0.5 cm s^{-1} through a liquid of density 0.8gcm^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of air.

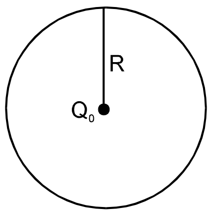
5. Two rods 'A' & 'B' of equal free length hang vertically 60 cm apart and support a rigid bar horizontally. The bar remains horizontal when carrying a load of 5000 kg at 20 cm from 'A'. If the stress in 'B' is 50 N/mm^2, find the stress in 'A' and the areas of 'A' and 'B'. Given $Y_B = 9 \times 10^4$ N/mm^2, $Y_A = 2 \times 10^5$ N/mm^2, g = 10 m/sec^2

6. A vertical rod 2 m long, fixed at the upper end, is 13 cm^2 in area for '1m' and 20 cm^2 in area for 1 m. A collar is attached to the free end. Through what height can a load of 100 kg fall on to collar to cause maximum stress of 50 N/mm^2. Y = 200000 N/mm^2. (g = 9.8 m/s^2)

7. A cylinder of 150 mm radius rotates concentrically inside a fixed cylinder of 155 mm radius. Both cylinders are 300 mm long. Determine the viscosity of the liquid which fills the space between the cylinders if a torque of 0.98 N-m is required to maintain an angular velocity of 60 r.p.m.



8. In a ring having linear charge density λ, made up of wire of cross-section area A, young modulus y, a charge Q_0 is placed at it's centre. If initial radius is 'R', then find out change in radius



9. A thin rod of negligible mass and area of cross-section 4×10^{-6} m^2, suspended vertically from one end has a length of 0.5 m at 10°C. The rod is cooled at 0°C, but prevented from contracting by attaching a mass at the lower end. Find **[JEE - 1997]**
(i) This mass and (ii) The energy stored in the rod.
Given for this rod, Y = 10^{11} Nm^{-2}, coefficient of linear expansion = 10^{-5} K^{-1} and g = 10ms^{-2}.

10. A long cylinder of radius R_1, is displaced along its axis with a constant velocity V_0 inside a stationary co-axial cylinder of radius R_2. The space between the cylinders is filled with viscous liquid .Find the velocity of the liquid as a function of the distance r from the axis of the cylinders. The flow is laminar.

11. A fluid with viscosity η fills the space between two long co-axial cylinders of radii R_1 and R_2 with $R_1 < R_2$. The inner Cylinder is stationary while the outer one is rotated with a constant angular velocity ω_2. The fluid flow is laminar. Taking into account that the friction force acting on a unit area of a cylindrical surface of radius r is defined by the formula $\sigma = \eta r(\partial\omega/\partial r)$, find:
(a) the angular velocity of the rotating fluid as a function of radius r;
(b) the moment of the friction forces acting on a unit length of the outer cylinder.

12. A tube of length ℓ and radius R carries a steady flow fluid whose density is ρ and viscosity η. The fluid flow velocity depends on the distance r from the axis of the tube as $v = v_0(1 - r^2/R^2)$ Find:
(a) the volume of the fluid flowing across the section of the tube per unit time;
(b) the kinetic energy of the fluid within the tube's volume;
(c) the friction force exerted on the tube by the fluid;
(d) the pressure difference at the ends of the tube.

HLP Answers

1.	88.632cm	**2.**	3.77×10^4 N	**3.**	20 cm/s	**4.**	35.55 poise
5.	$\frac{1000}{9}$ N/mm², 300 mm², $\frac{1000}{3}$ mm²			**6.**	1.33 cm	**7.**	$\eta = 0.77$ N-sec/m².
8.	$\Delta R = \frac{k\lambda Q_0}{AY}$	**9.**	(i) 4.0kg (ii) 0.001 J			**10.**	$v = v_0 \frac{\ell n(r/R_2)}{\ell n(R_1/R_2)}$

11. (a) $\omega = \omega_2 \frac{R_1^2 R_2^2}{R_2^2 - R_1^2}\left(\frac{1}{R_1^2} - \frac{1}{r^2}\right)$; (b) $N = 4\pi\eta\omega_2 \frac{R_1^2 R_2^2}{R_2^2 - R_1^2}$

12. (a) $Q = 1/2\pi v_0 R^2$; (b) $T = 1/6\ \pi\ell R^2 \rho v_0^2$; (c) $F_{fr} = 4\pi\eta\ell v_0$; (d) $\Delta p = 4\pi\eta\ell v_0/R^2$

HINT & SOLUTION OF ELASTICITY & VISCOSITY
EXERCISE-1
PART - I

A-1. (a) $\sigma = \frac{F}{A} = \frac{3\times10^4}{3.6\times10^{-4}} = \frac{30}{36}\times10^8 = \frac{5}{6}\times10^8 < 7.7\times10^8\ N/m^2$

hence it will not break.

(b) $\Delta\ell = \frac{FL}{AY} = \frac{3\times10^4\times20\times10^{-2}}{3.6\times10^{-4}\times1.5\times10^{10}} = \frac{40\times10^{-4}}{3.6} = \frac{10}{9}\times10^{-3}\ m$

A-2. $\Delta\ell_S = \frac{F\ell}{Y_S A}$ $\Delta\ell_C = \frac{F\ell}{Y_C A}$

$\Delta\ell_S - \Delta\ell_C = 0.5$

$\frac{F\ell}{A}\left(\frac{1}{Y_s} - \frac{1}{Y_c}\right) = 0.5$

$\frac{F\ell}{A} = \frac{Y_c Y_s \times 0.5}{(Y_c - Y_s)}$

$\frac{F\ell}{A} = 15\times10^{11}$

$\Delta\ell_S = \left(\frac{F\ell}{A}\right)\frac{1}{Y_s} = \frac{15\times10^{11}}{2\times10^{12}} = 0.75\ cm$

$\Delta\ell_C = \left(\frac{F\ell}{A}\right)\times\frac{1}{Y_c} = \frac{15\times10^{11}}{12\times10^{11}} = 1.25\ cm$

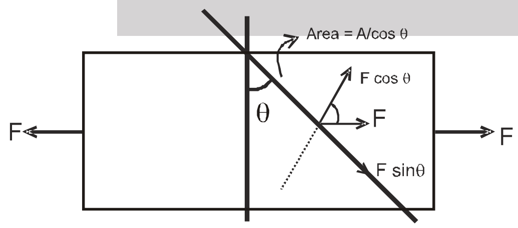
A-3. $a = \frac{40}{12} = \frac{10}{3}\ m/s^2$

$T_1 = \frac{40}{3}\ N$ $T_2 = \frac{80}{3}\ N$

strain in wire 1 = $\frac{40}{3\times5\times10^{-7}\times2\times10^{11}} = \frac{4}{3}\times10^{-4}$

strain in wire 2 = $\frac{80}{3\times5\times10^{-7}\times2\times10^{11}} = \frac{8}{3}\times10^{-4}$

B-1.



(a) tensile stress = $\frac{F\cos\theta}{A/\cos\theta} = \frac{F\cos^2\theta}{A}$

(b) shearing stress = $\frac{F\sin\theta}{A/\cos\theta} = \frac{F}{A}\sin\theta\cos\theta$

(c) for max. tensile stress

$\theta = 0^o$

(d) for max. shearing stress

$\theta = 45^o$

C-1. $B = \dfrac{P}{\dfrac{\Delta V}{V}} = \dfrac{100\text{atm}}{10 \quad \times 10^{-6}} = 10^7$ atm

D-1. $K = \dfrac{AY}{\ell} = \dfrac{1\times10^{-4}\times1\times10^{11}}{0.2} = 5 \times 10^7$

$U = \dfrac{F^2}{2K} = \dfrac{(5\times9.8)^2}{2\times5\times10^7} = \dfrac{49\times49}{10^8} = 2.40 \times 10^{-5}$ J

D-2. $W = \dfrac{1}{2}(F_2 x_2 - F_1 x_1)$

$= \dfrac{1}{2}(4 \times 9.8 \times 10^{-3} - 2 \times 9.8 \times 0.6 \times 10^{-3})$

$= 2 \times 9.8 \times 10^{-3} \times 0.7$

$= 13.72 \times 10^{-3}$ J

E-1. $v = \dfrac{2}{9\eta} r^2.(\rho_0 - \rho_w)\, g$

= 180 m/sec.

$h = \dfrac{32400}{2\times9.8} = \dfrac{81}{49}\times10^3$ m

PART - II

A-1. d = 4mm

$Y = 9 \times 10^{10}$ N/m^2

$\dfrac{F}{A} = Y\dfrac{\Delta\ell}{\ell}$

$F = AY\dfrac{\Delta\ell}{\ell}$

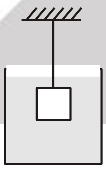
$= \pi(2\text{x}10^{-3})^2 \text{ x } 9 \times 10^9 \text{ x } \dfrac{1}{100}$

$= \pi \text{ x } 4 \text{ x } 10^{-6} \text{ x } 9 \times 10^7$

$= 360\,\pi$ N

A-2.

$\Delta\ell_{air} = L_a$

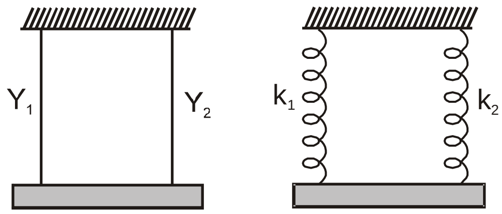


$\Delta\ell_{water} = L_w$

$L_a = \dfrac{WL}{YA}$ $\quad L_w = \dfrac{\left[W - \dfrac{W}{\rho_o}\rho_w\right] L}{YA} = \dfrac{W[1 - \dfrac{\rho_w}{\rho_o}]\; L}{YA}$

$\dfrac{L_a}{L_w} = \left[1 - \dfrac{\rho_w}{\rho_o}\right] \Rightarrow \dfrac{\rho_o}{\rho_w} = \dfrac{L_a}{L_a - L_w}$

A-3.

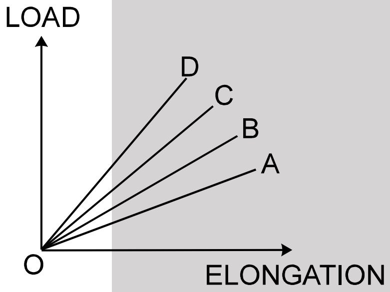


$K_{eq} = K_1 + K_2$

$$\frac{Y2A}{\ell} = \frac{Y_1A}{\ell} + \frac{Y_2A}{\ell}$$

$$Y = \frac{Y_1 + Y_2}{2}$$

A-4.



$$\frac{F/A}{\Delta\ell/\ell} = Y$$

$$\frac{F}{\Delta\ell} = \frac{Y\pi r^2}{\ell}$$

$$\Rightarrow \quad \frac{F\ell}{Y\pi} \times \frac{1}{\Delta\ell} = r^2$$

$\Rightarrow$ Y & ℓ are same for all then

For same load $r \propto \frac{1}{\sqrt{\Delta\ell}}$

B-1. $F = \eta A \frac{x}{h} = 0.4 \times 10^{11} \times 1 \times .005 \times \frac{.02 \times 10^{-2}}{1} = 4 \times 10^4$ N

C-1. $\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{1 \times 10^5}{1.25 \times 10^{11}} = 8 \times 10^{-7}$

D-1. $V = 1/2\ K(2)^2$
$V' = 1/2\ K(10)^2$
then $V' = 25V$

D-2. $K = \frac{AY}{\ell}$, $\quad K' = \frac{4AY}{\ell/2} = 8K$

$$\frac{U}{2} = \frac{\frac{1}{2} \times 8K \times \Delta\ell^2}{\frac{1}{2} \times K \times \Delta\ell^2} \Rightarrow \quad U = 16 \text{ J}$$

E-1. (i) $v = 5 \times 10^{-4}$ m/s

$v = \frac{2}{9\eta} r^2 \rho g$

$r^2 = \frac{5 \times 9 \times 18 \times 10^{-5} \times 10^{-4}}{2 \times 900 \times 10} = 9 \times 10^{-12}$

$r = 3 \times 10^{-6}$ m

(ii) $v \propto r^2$

$\frac{v_1}{v} = \frac{r_1^2}{r^2} = \frac{1}{4}$,

$v_1 = \frac{5 \times 10^{-4}}{4} = 1.25 \times 10^{-4}$ m/sec

E-2. $v = \frac{2}{9\eta} r^2 . (\rho_0 - \rho_w)\, g$

E-3. Velocity increases till $F_B + F_V = mg$

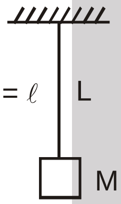
F_B = Bouyancy force

F_V = Viscous force

E-4. There will not be any viscous force so velocity will keep on increasing.

PART - III

1. $\Delta\ell = \ell$



loss in PE = $Mg\ell$

Elastic PE = $\frac{1}{2} Kx^2$

$= \frac{1}{2} \frac{Mg}{A} \times \frac{\ell}{L} \times AL$

$= MgL/2$

Heat = MgL – Mg L/2

= Mg L/2

EXERCISE-2

PART – I

1. $\frac{F}{A} = Y \frac{\Delta\ell}{\ell}$ If Y & $\frac{\Delta\ell}{\ell}$ are constant

$F = AY \frac{\Delta\ell}{\ell} \Rightarrow F \propto A \Rightarrow F' = 4F$

2. $\frac{p_1}{p_2} = \frac{m_1 v_1}{m_2 v_2}$, $m \propto r^3$, $v \propto r^2 \Rightarrow p \propto r^5$ then $\frac{p_1}{p_2} = \frac{1}{32}$

3. $\frac{F}{A} = \eta \frac{x}{h}$

$$\frac{500}{4 \times 16 \times 10^{-4}} = 2 \times 10^{6} \frac{x}{4 \times 10^{-2}} \Rightarrow x = \frac{5 \times 10^{-2}}{32} m = 0.156 \text{ cm}$$

4. $\ell_B = 2m$ $\quad \ell_S = L$

$A_B = 2\ cm^2$ $\quad A_S = 1\ cm^2$

$\Delta\ell_B = \Delta\ell_S$

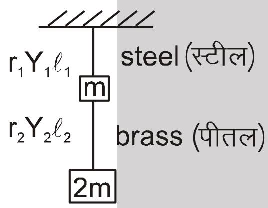
$$\frac{F}{A_B}\frac{\ell_B}{Y_B} = \frac{F}{A_S}\frac{\ell_S}{Y_S}$$

$$L = \frac{A_S Y_S}{A_B Y_B}\ell_B = \frac{1}{2} \times \frac{2 \times 10^{11}}{1 \times 10^{11}} \times 2 = 2$$

6. 46.4×10^{-6} atm $= \frac{1}{B}$

$$B = \frac{1}{46.4 \times 10^{-6}} \Rightarrow B = \frac{P}{\Delta v / v} \Rightarrow \frac{\Delta v}{v} = \frac{\Delta p}{B} = 46.4 \times 10^{-6}$$

7.



$\frac{r_1}{r_2} = b$

$\frac{\ell_1}{\ell_2} = a$

$\frac{Y_1}{Y_2} = c$

$$\Delta\ell_1 = \frac{(3\ mg)\ \ell_1}{A_1 Y_1}$$

$$\Delta\ell_2 = \frac{(2mg)\ \ell_2}{A_2 Y_2}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{3\ell_1}{2\ell_2\ A_1 Y_1} \times A_2 Y_2 = \frac{3}{2}\frac{a}{b^2 c} = \frac{3a}{2b^2 c}$$

8. depth = 200 m

$$\frac{\Delta V}{V} = \frac{0.1}{100} = 10^{-3}$$

density $= 1 \times 10^3$

g = 10

$$B = \frac{\Delta p}{\Delta v / v} = \frac{hg\rho}{\Delta v / v} \Rightarrow B = 200 \times 10 \times 10^3 \times 1000 = 2 \times 10^9$$

9. $\frac{r_1}{r_2} = \frac{1}{2}$

PE (per unit volume) $= \frac{1}{2Y}\left(\frac{F}{A}\right)^2$

PE $\propto 1/A^2$

$\frac{PE_1}{PE_2} = \frac{A_2^2}{A_1^2} = 16 : 1$

10. Magnitude of viscous force is equal to weight and in later case it will act downward.

PART - II

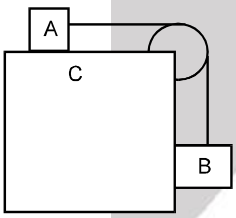
1. $\frac{\left(\frac{\sigma}{2}\right)^2}{2Y}\,10x + \frac{\sigma^2}{2Y}\,(1 - x)\,5 = \frac{40}{100}\left[\frac{\sigma^2}{2Y}10 \times 1\right]$

$2.5x + 5 - 5x = 4$
$2.5x = 1$
$x = 40cm$

2. $\delta = \frac{Fdx}{AY}$, $\Delta\ell = \int \frac{Fdx}{AY}$

$= \int_0^{10} \frac{20 \times 10^3 dx}{\left(1 + \frac{x^2}{100}\right) \times 2 \times 10^7}$ = 0.008 cm

3.



$kx = \mu M_A g$ ...... (i)
$kx = M_B g$ ...... (ii)

$\Rightarrow$ $x = \frac{M_B g}{K} = \frac{1}{100}$

From eq. (i) & (ii)

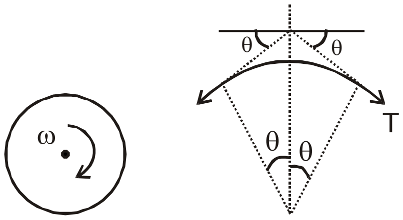
$0.2 M_A = M_B$

$M_A = 10$ kg

$E = \frac{1}{2}Kx^2 = \frac{1}{2} \times 2000 \times \left(\frac{1}{100}\right)^2$

$E = 10 \times 10^{-2}$

4.



$$2T\sin\theta = \left(\frac{m}{2\pi R}R.2\theta\right)\omega^2 R'$$

For small 'θ' -

$$T = \frac{m\omega^2 R'}{2\pi}$$

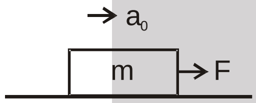
$$\frac{T}{A} = \left(\frac{2\pi R' - 2\pi R}{2\pi R}\right)Y$$

$$\frac{m\omega^2 R'}{2\pi A} = \frac{\Delta R}{R}Y, \text{ If } R' \simeq R$$

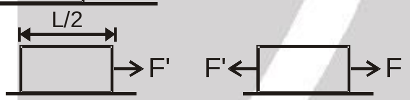
$$\Delta R = \frac{m\omega^2 R^2}{2\pi AY} \text{ but } m = A\,2\pi R\,\rho$$

$$\Delta R = \frac{A 2\pi R\rho\omega^2 R^2}{2\pi AY} = \frac{\rho\omega^2 R^3}{Y}$$

5.

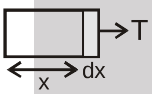


(a)



$$F' = \frac{m}{2}a_0\ ,\ \sigma = \frac{F'}{S} = \frac{ma_0}{2S} = \frac{\rho LSa_0}{2S} = \frac{\rho La_0}{2}$$

(b)



$$T = \frac{mx}{L}a_0$$

$$\delta = \frac{Tdx}{SY}$$

$$\Delta\ell = \int_0^L \frac{Tdx}{SY} = \int_0^L \frac{mxa_0}{SYL}dx = \frac{ma_0L}{2SY} = \frac{\rho LSa_0L}{SYL} = \frac{\rho L^2 a_0}{2Y}$$

6. $F = \eta A\frac{\Delta V}{\Delta Z}$

$$9.8 = 5\times10^{-3}\pi \times 796\times10^{-3} \times 200 \times 10^{-3} \times \frac{v}{2\times10^{-3}}$$

$v = 7.841$ m/s

7. $F = \eta A\frac{\Delta V}{\Delta Z}$ where $F = mg\sin\theta = 130 \times \frac{5}{13}$

$$130 \times \frac{5}{13} = \eta \times1\times1\times \frac{0.5}{0.15\times10^{-2}}$$

$$\eta = \frac{50\times0.15\times10^{-2}}{0.5} = 0.15$$

8. $d\tau = r\eta\left(2\pi r dr \dfrac{\omega r}{h}\right)$

$\tau = \int_0^r \dfrac{\eta}{h}\ 2\pi\omega r^3 dr$

$\tau = \int_0^{d/2} \dfrac{\eta\omega}{h} 2\pi\ \ r^3 dr$

$\tau = \left[\dfrac{2\pi\eta\omega}{h}\ \ \dfrac{r^4}{4}\right]_0^{d/2}$

$\tau = \dfrac{2\pi\eta\omega}{h}\ \ \dfrac{d^4}{4 \times 16}$

$\tau = \left(\dfrac{\pi\eta\omega d^4}{32h}\right)$

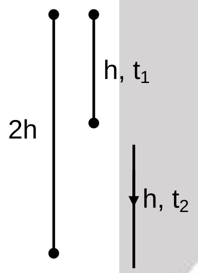
PART - III

1. Stress in wire B $= \dfrac{mg}{3\pi r_B^2}$

Stress in wire A $= \dfrac{4mg}{3\pi r_A^2}$

if $\dfrac{mg}{3\pi r_B^2} = \dfrac{4mg}{3\pi r_A^2}$ either wire will break.

2.



Time t_1 will be more then t_2 because speed increases from zero to terminal speed in t_1 duration and ball covers a distance h.

Work done against viscous force depends on magnitude of viscous force and displacement ball.

viscous force increases from zero to maximum value and then remains constant

3. $W = -\Delta U = \dfrac{1}{2}Kx^2 = \dfrac{1}{2}\ \dfrac{AY}{L}\ell^2$

PART - IV

1. $K = \dfrac{AY}{\ell} = \dfrac{4 \times 10^{-4} \ \times\ 2\ \times\ 10^{10}}{4} = 2 \times 10^6$

$\omega = \sqrt{\dfrac{K}{m}} = 100$

2. $W(h + x) = 1/2\ kx^2$

$100(0.99 + x) = \frac{1}{2} \times 2 \times 10^6 \times x^2$

$10^4 x^2 - x - 0.99 = 0$

$100x(100x - 1) + 0.99(100x - 1) = 0$

$x = \frac{1}{100}$ m = 1 cm

3. $x = \frac{PL}{AY}$

$\sigma = \frac{P}{A} = \frac{xY}{L} = \frac{10^{-2} x\ 2\ x\ 10^{10}}{4} = 5 \times 10^7\ N/m^2$

4. $K_1 = 10^6$, $K_2 = 2 \times 10^6$

$K_{eq} = \frac{2 \times 10^6 x 10^6}{3 \times 10^6} = \frac{2}{3} \times 10^6$

$\omega = \sqrt{\frac{2 \times 10^6}{3 \times 600}} = \frac{100}{3}$

5. Total weight = 1000 + w

weight on each rod $= \frac{1000 + w}{4}$

stress $= \frac{1000 + w}{4 \times 4 \times 10^{-4}} = 9 \times 10^6 \Rightarrow$ w = 14400 – 1000 = 13400 N

No. of persons are $= \frac{1340}{50} = 26$

6. $F = \eta A \frac{du}{dy}$

as $u = C_1 y + C_2$

at y = 0 , u = 0 hence $C_2 = 0$

$\frac{du}{dy} = C_1$

$F = \eta A C_1$

$2 = 10^{-2} \times 1\ C_1$

$C_1 = 200$

7. $u = c_1 y + c_2$

$c_1 = 200$, $c_2 = 0$

$u = 200 \times 2 \times 10^{-2} = 4$ m/sec

8. y = 0 , u = 0 $C_3 = 0$

y = 2 cm , u = 2 m/sec

$2 = C_1\ 4 \times 10^{-4} + C_2\ 2 \times 10^{-2}$ ....(1)

$\frac{du}{dy} = 2C_1 y + 2$

$F = \eta A \frac{du}{dy}$

at y = 2cm, F = 2 N

$2 = 10^{-2} \times 1 \times [2 \times 2 \times 10^{-2} C_1 + C_2]$

$4 \times 10^{-4} C_1 + 10^{-2} C_2 = 2$ ....(2)

$4 \times 10^{-4} C_1 + 2 \times 10^{-2} C_2 = 2$ ....(1)

on solving

$C_2 = 0$ & $C_1 = 5000$

9. $\frac{du}{dy} = \frac{F}{\eta A} = \frac{2}{10^{-2} \times 1} = 200$

10. $\frac{du}{dy} = 2C_1 y + C_2$

at y = 0, $\frac{du}{dy} = C_2 = 0$

EXERCISE-3

PART - I

1. Terminal velocity $v_T = \frac{2r^2 g}{9\eta}(\rho_s - \rho_L)$

and viscous force $F = 6\pi\eta r v_T$

Viscous force is the only dissipative force. Hence

$\frac{dQ}{dt} = Fv_T = (6\pi\eta r v_T)(v_T) = 6\pi\eta r v_T^2$

$= 6\pi\eta r\left\{\frac{2}{9}\frac{r^2 g}{\eta}(\rho_s - \rho_L)\right\}^2 = \frac{8\pi g^2}{27\eta}(\rho_s - \rho_L)^2 r^5 = \frac{dQ}{dt} \propto r^5$

2. $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{yA/\ell}{m}} = \sqrt{\frac{yA}{\ell m}} \Rightarrow \sqrt{\frac{(n \times 10^9) \times (4.9 \times 10^{-7})}{1 \times 0.1}} = 140 \Rightarrow n = 4.$

3. In equilibrium,

$mg = qE$

In absence of electric field,

$mg = 6\pi\eta rv$

$\Rightarrow qE = 6\pi qrv$

$m = \frac{4}{3}\pi R r^3 d. = \frac{qE}{g}$

$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$

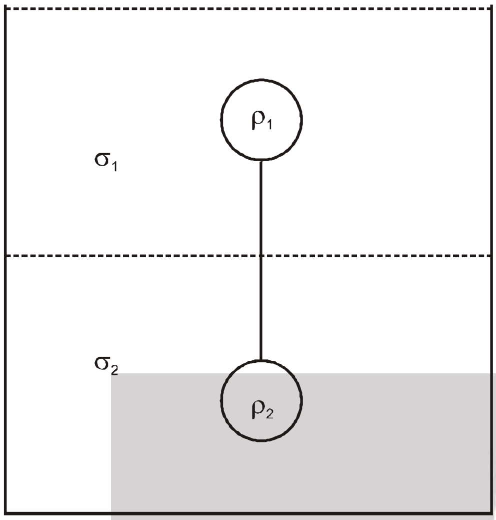
After substituting value we get,

$q = 8 \times 10^{-19}$ C **Ans.**

4. $Y = \frac{\left(\frac{F}{A}\right)}{\frac{\Delta\ell_1}{L}}$ ...(i) $Y = \frac{\left(\frac{F}{4A}\right)}{\frac{\Delta\ell_2}{2L}}$ ...(ii)

$\frac{\Delta\ell_1}{\Delta\ell_2} = 2$

6.



For floating

$(\rho_1 + \rho_2)V = (\sigma_1 + \sigma_2)V$

$\rho_1 + \rho_2 = \sigma_1 + \sigma_2$

since strings in taut so

$\rho_1 < \sigma_1$ $\qquad$ $\rho_2 > \sigma_2$

$$V_P = \frac{2}{9}\frac{r^2(\sigma_2 - \rho_1)g}{\eta_2}$$

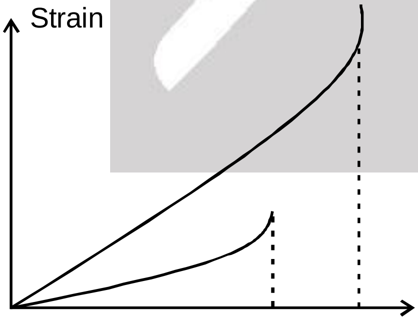
$$V_Q = \frac{2}{9}\frac{(\sigma_1 - \rho_2)g}{\eta_1}$$

since $\sigma_2 - \rho_1 = -(\sigma_1 - \rho_2)$

$$\left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2}$$

$\vec{V}_P \cdot \vec{V}_Q < 0$ because V_P and V_Q are opposite

7.



Breaking stress of P is more than Q so P is more ductile

strain = $\frac{1}{Y}$ stress

$Y_p < Y_Q$

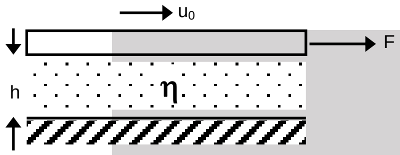
8. $6\pi\eta rv + \rho_L Vg = \rho_0 Vg$

$$\frac{v_P}{v_Q} = \frac{(\rho_P V_P - \rho_L V_p)g}{6\pi\eta_p r_p} \times \frac{6\pi\eta_Q r_Q}{(\rho_Q V_Q - \rho_L V_Q)}$$

$$= \frac{r_p^3(8-0.8)}{\eta_p . r_p (8-1.6)} \times \frac{r_Q.\eta_Q}{r_Q^3}$$

$$= \left(\frac{r_p}{r_Q}\right)^2 \times \left(\frac{\eta_Q}{\eta_p}\right) \times \left(\frac{7.2}{6.4}\right) = 4 \times \frac{7.2}{6.4} \times \frac{2}{3} = 3$$

9.



$$F = \eta A \frac{dv}{dy}$$

$$= \eta A \frac{u_0}{h}$$

10. $v_0 = \frac{J}{m} = \frac{1}{0.4} = \frac{5}{2}$ m/s

$$s = v_0 \int_0^{\tau} e^{-t/\tau} dt$$

$$= v_0 \tau (1 - e^{-1})$$

$$= \left(\frac{5}{2}\right) (4)\ (0.63) = 6.30 \text{ m}$$

PART - II

1. Retarding force acting on a ball falling into a viscous fluid

$F = 6\pi\eta Rv$

where R = radius of ball,

v = velocity of ball,

and η = coefficient of viscosity

$\therefore$ $F \propto R$ and $F \propto v$

2. Young's modulus = $\frac{\text{Strees}}{\text{Strain}}$

$\Rightarrow$ Strain = $\frac{S}{Y}$

$$\frac{\text{Energy stored in wire}}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

3. Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where ρ = density of substance of body,

σ = density of liquid.

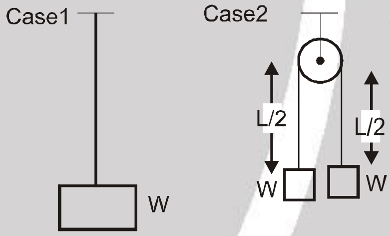
From given data

$$\frac{v_T(Ag)}{v_T(Gold)} = \frac{\rho_{Ag} - \sigma_l}{\rho_{Gold} - \sigma_l}$$

$\Rightarrow$ $v_T\ (Ag) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2 = \frac{9}{18} \times 0.2$

= 0.1 m/s

4. Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.



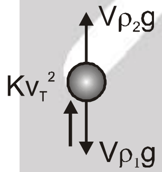
Then for 1st case Y = $\frac{W/A}{\ell/L}$

For 2nd case, Y = $\frac{W/A}{2\ell'/L}$

$\therefore$ $l' = l/2$

So, total elongation of both sides = $2\ell' = \ell$

5. The forces acting on the ball are gravity force, buoyancy force and viscous force. When ball acquires terminal speed, it is in dynamic equilibrium, let terminal speed of ball is v_T . So,



$V\rho_2 g + kv_T^2 = V\rho_1 g$ $\qquad v_T = \sqrt{\frac{V(\rho_1 - \rho_2)g}{k}}$

6. F = $\frac{Y\ \ A\ \ x}{\ell}$

and $F_2 = \frac{Y\ \ (3A\ )\ \ x}{(\ell/3)}$ = 9 F

7. $V\rho g = 6\pi\eta rv + v\rho_\ell g$

$Vg(\rho - \rho_\ell) = 6\pi\eta rv$

$Vg(\rho - \rho_\ell') = 6\pi\eta' rv'$

$$V'\ \eta' = \frac{(\rho-\rho_\ell')}{(\rho-\rho_\ell)} \times v\eta$$

$$V' = \frac{(\rho-\rho_\ell')}{(\rho-\rho_\ell)} \times \frac{v\eta}{\eta'}$$

$$= \frac{(7.8-1.2)}{(7.8-1)} \times \frac{10\times 8.5\times 10^{-4}}{13.2}$$

$v' = 6.25 \times 10^{-4}$ cm/s.

8. $\frac{P}{\alpha\Delta\theta} = Y$

$P = Y\alpha\Delta\theta = 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^{8}$ Pa

9. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$ $\qquad \Delta\ell = \frac{Mg\ell}{AY}$

$$\frac{T_M}{T} = \sqrt{\frac{\ell+\Delta\ell}{\ell}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{\Delta\ell}{\ell}$$

$$\frac{1}{y} = \left(\left(\frac{T_M}{T}\right)^2 - 1\right) = 1 + \frac{Mg}{AY}$$

$$\frac{1}{y} = \left(\left(\frac{T_M}{T}\right)^2 - 1\right)\frac{A}{Mg}$$

10. $\Delta P = \frac{mg}{a}$

$$K = -\frac{\frac{mg}{A}}{\frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3}}$$

$$\frac{dr}{r} = -\frac{mg}{3KA}$$

HIGH LEVEL PROBLEMS (HLP)

SUBJECTIVE QUESTIONS

1. $\rho = 7.6\ g/cm^3$

$\ell + \Delta\ell = 90$ cm

$\Delta\ell = \dfrac{w\ell}{AY}$ ....(1)

$\Delta\ell - 0.18 = \dfrac{\left(w - \dfrac{w}{\rho_0}\rho_\ell\right)\ell}{AY}$ ....(2)

$\dfrac{\Delta\ell - 0.18}{\Delta\ell} = 1 - \dfrac{\rho_\ell}{\rho_0}$

$1 - \dfrac{0.18}{\Delta\ell} = 1 - \dfrac{1}{7.6}$

$\Delta\ell = 7.6 \times 0.18$

then $\ell + \Delta\ell = 90$

$\ell = 90 - 7.6 \times 0.18 = 88.632$ cm

2. r = 2 mm

$\left(\dfrac{F}{2A}\right)_{max} = 1.5 \times 10^9$

$F = 1.5 \times 10^9 \times A \times 2$

$= 1.5 \times 10^9 \times \pi(2 \times 10^{-3})^2 \times 2$

$= 18840 \times 2 = 3.77 \times 10^4$ N

3. $\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi . r^3 \times 8$

$R = 2r$

$V \propto r^2$

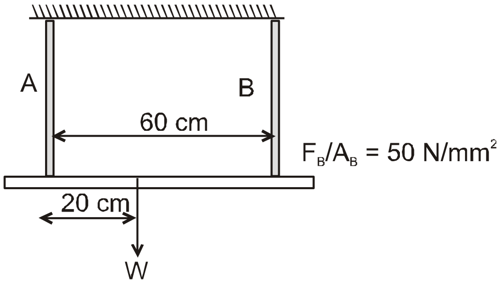
Hence 20 cm/sec.

4. $v = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho_0 - \rho_a)\ g$

$0.5 = \dfrac{2}{9\eta} \times 0.8 \times 100$

$\eta = \dfrac{320}{9} = 35.55$ poise **Ans.**

5.



$Y_A = 2 \times 10^5$ N/mm^2
$Y_B = 9 \times 10^4$ N/mm^2
$F_A \times 20 = F_B \times 40$
$F_A = 2F_B$
$F_A + F_B = 5000$ g

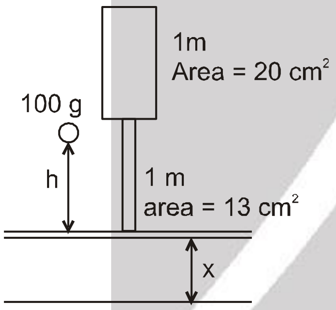
$F_B = \frac{5000\ g}{3}$, $F_A = \frac{10000\ g}{3}$

$\sigma_B = 50 = \frac{F_B}{A_B}$ $\qquad A_B = \frac{50 \times 10^3}{3 \times 50} = \frac{1000}{3}$ mm^2

$\Delta\ell = \frac{F_B \ell}{A_B Y_B} = \frac{F_A \ell}{A_A Y_A}$, $\qquad \sigma_A = \frac{F_A}{A_A} = \frac{Y_A}{Y_B}\sigma_B = \frac{20}{9} \times 50 = \frac{1000}{9}$ N/mm^2

$\sigma_A = \frac{1000}{9} = \frac{10^5}{3\ \ A_A}$ $\qquad A_A = 300$ mm^2

6.



$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$, $k_1 = \frac{A_1 Y}{\ell}$, $k_2 = \frac{A_2 Y}{\ell}$, $\ell = 1$ m

$k = \frac{k_1 k_2}{k_1 + k_2} = \frac{A_1 A_2}{(A_1 + A_2)} \frac{Y}{\ell}$

$k = \frac{52}{33} \times 10^8$

$F = 50 \times 10^6 \times 13 \times 10^{-4} = 650 \times 10^2$ N
$F = kx$

$x = \frac{165}{4} \times 10^{-5}$ m $\qquad mg(h + x) = \frac{1}{2}kx^2 = \frac{F^2}{2k}$

on solving $\qquad h = 1.33$ cm

7. $\tau = Fr$

$$F = \frac{\eta(2\pi r_i) \times \omega r_i \times \ell}{r_0 - r_i}$$

$$\Rightarrow \quad \tau = 2\pi\eta r_i^3 \times \frac{\omega \times \ell}{(r_0 - r_i)}$$

on solving and putting values

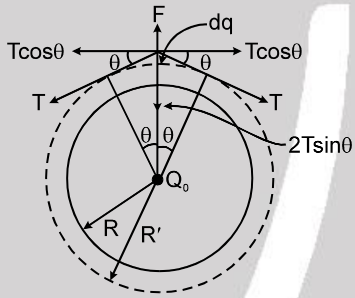
$\eta = 0.77$ N-s/m^2

8. Considering an element of angular width 2θ -

$dq = \lambda R' \cdot 2\theta \quad \Rightarrow \quad F = 2T \sin\theta$

$$\frac{kdq.\ Q_0}{R'^2} = 2T\sin\theta \quad \Rightarrow \quad \frac{k\lambda R' \times 2\theta.\ Q_0}{R'^2} = 2T\sin\theta$$

if θ is small, then $\sin\theta = \theta$



further $\quad \frac{k\lambda Q_0}{R'} = T$

But $\quad Y = \frac{\text{stress}}{\text{strain}}$, strain $= \frac{2\pi(R' - R)}{2\pi R} = \frac{R' - R}{R}$

$$Y = \frac{T}{a(R' - R)} \Rightarrow R' - R = \frac{k\lambda Q_0}{R'AY}$$

$$Y = \frac{T}{A\frac{\Delta R}{R}} \Rightarrow \Delta R = \frac{TR}{AY} = \frac{k\lambda Q_0 R}{R'AY} = (R \simeq R')$$

9. $\frac{mg}{A} = Y\alpha\Delta\theta \Rightarrow m = \frac{Y\alpha\Delta\theta A}{g} = 4$kg

$$U = \frac{F^2 L}{2AY} = 10^{-3} \text{ J}$$

HANDOUT–3 ELASTICITY AND VISCOSITY

ELASTICITY AND PLASTICITY

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming force : An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic body : A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fiber is the nearest approach to the perfectly elastic body.

Perfectly Plastic body : A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, so the concept of perfectly plastic body is also only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity : In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighboring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are minimum.
On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

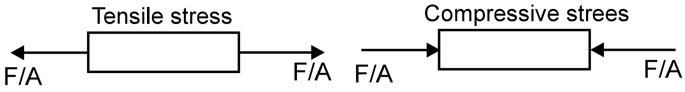
When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body is called stress.

$$\text{stress} = \frac{\text{restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2. There are three types of stress

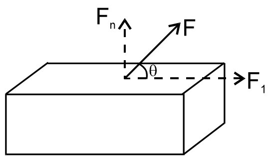
1. Longitudinal or Normal stress

When object is one dimensional then force acting per unit area is called longitudinal stress.
It is of two types : (a) compressive stress (b) tensile stress



Examples :

(i) Consider a block of solid as shown in figure. Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components :
$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.



(b)

$\therefore$ Normal (tensile) stress $= \dfrac{F_n}{A} = \dfrac{F\sin\theta}{A}$

2. Tangential or shear stress

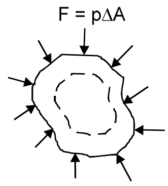
It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to shown in figure above.

Tangential (shear) stress = $\frac{F_t}{A} = \frac{F\cos\theta}{A}$

The effect of stress is to produce distortion or a change in size, volume and shape (i.e. configuration of the body).

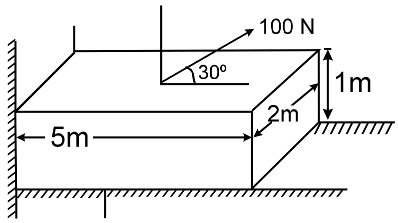
3. Bulk stress

When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon the homogeneity of body.



Solved Example

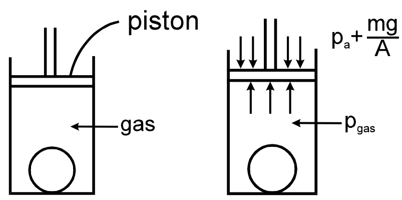
Example 1. Find out longitudinal stress and tangential stress on a fixed block.



Solution : Longitudinal or normal stress $\Rightarrow$ $\sigma_l = \frac{100\sin 30^\circ}{5\times 2}$ = 5 N/m^2

Tangential stress $\Rightarrow$ $\sigma_t = \frac{100\ \cos\ 30^\circ}{5\times 2} = 5\sqrt{3}\,\text{N/m}^2$

Example 2. Find out Bulk stress on the spherical object of radius $\frac{10}{\pi}$ cm if area and mass of piston is 50 cm^2 and 50 kg respectively for a cylinder filled with gas.



Solution : $p_{gas} = \frac{mg}{A} + p_a = \frac{50\times 10}{50\times 10^{-4}}$ + 1 × 10^5 = 2 × 10^5 N/m^2

Bulk stress = p_{gas} = 2 × 10^5 N/m^2

STRAIN

The ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body is called strain,

i.e. Strain, $\in = \frac{\text{change in configuration}}{\text{original configuration}}$

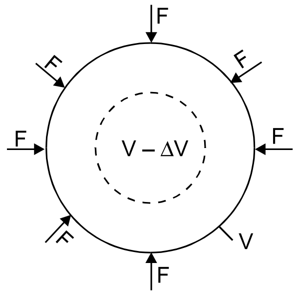
It has no unit

Types of strain : There are three types of strain

(i) Longitudinal strain : This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body. Consider a wire of length L **:** When the wire is stretched by a force F, then let the change in length of the wire is ΔL.

$\therefore$ Longitudinal strain, $\in_\ell = \dfrac{\text{change in length}}{\text{original length}}$ or Longitudinal strain $= \dfrac{\Delta L}{L}$

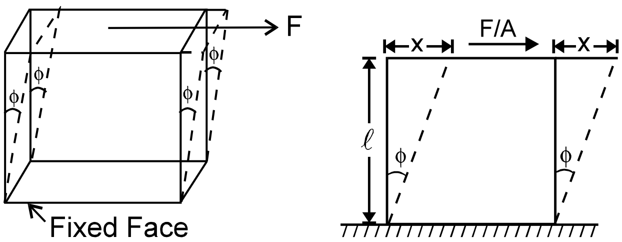
(ii) Volume strain : This type of strain is produced when the deforming force produces a change in volume of the body as shown in the figure. It is defined as the ratio of the change in volume to the original volume of the body.



If ΔV = change in volume V = original volume

$\in_V$ = volume strain = $\dfrac{\Delta V}{V}$

(iii) Shear Strain : This type of strain is produced when the deforming force causes a change in the shape of the body. It is defined as the angle ϕ through which a face originally perpendicular to the fixed face is turned as shown in the figure.



tan ϕ or $\phi = \dfrac{x}{\ell}$

HOOKE'S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain

or stress = constant × strain or $\dfrac{\text{stress}}{\text{strain}}$ = Modulus of Elasticity.

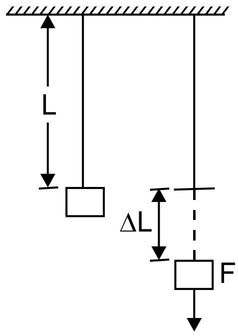
This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

Modulus of elasticity depends on the nature of the material of the body and is independent of its dimensions (i.e. length, volume etc.).

Unit : The SI unit of modulus of elasticity is Nm^{-2} or Pascal (Pa).

TYPES OF MODULUS OF ELASTICITY



Corresponding to the three types of strain there are three types of modulus of elasticity.

1. Young's modulus of elasticity (Y)
2. Bulk modulus of elasticity (K)
3. Modulus of rigidity (η).

1. Young's modulus of elasticity

It is defined as the ratio of the normal stress to the longitudinal strain.

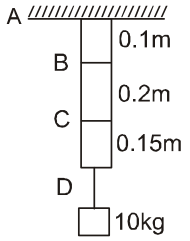
i.e. Young's modulus (Y) = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

Normal stress = F/A,

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

Example 3. Find out the shift in point B, C and D



$Y_{AB} = 2.5 \times 10^{10}\ N/m^2$

$Y_{BC} = 4 \times 10^{10}\ N/m^2$

$Y_{CD} = 1 \times 10^{10}\ N/m^2$

$A = 10^{-7}\ m^2$

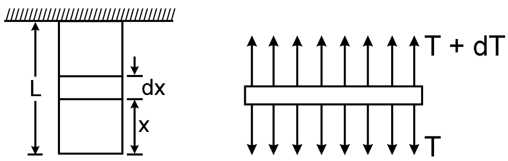
Solution : $\Delta L_B = \Delta L_{AB} = \frac{FL}{AY} = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3}$ m = 4mm

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \frac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}} = 4 \times 10^{-3} + 5 \times 10^{-3}$ = 9mm

$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \frac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}} = 9 \times 10^{-3} + 15 \times 10^{-3}$ = 24 mm

ELONGATION OF ROD UNDER IT'S SELF WEIGHT

Let rod is having self weight 'W', area of cross-section 'A" and length 'L'. Considering on element at a distance 'x' from bottom.



then $T = \frac{W}{L}x$

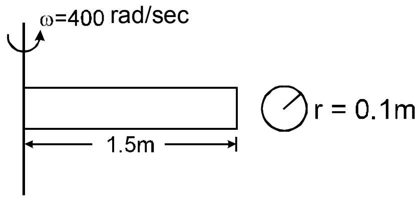
elongation in 'dx' element = $\frac{Tdx}{AY}$

Total elongation $s = \int_0^L \frac{Tdx}{AY} = \int_0^L \frac{Wx\ dx}{LAY} = \frac{WL}{2AY}$

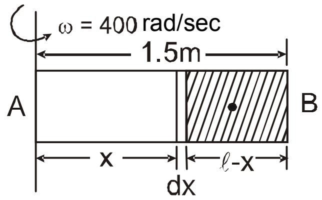
Note : One can do directly by considering total weight at C.M. and using effective length $\ell/2$.

Solved Example

Example 4. Given $Y = 2 \times 10^{11}$ N/m^2, $\rho = 10^4$ kg/m^3. Find out elongation in rod.



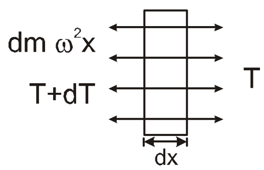
Solution : mass of shaded portion

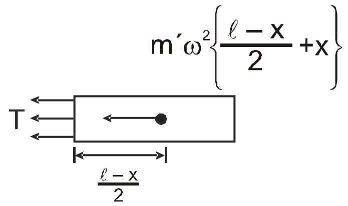


$m' = \frac{m}{\ell}(\ell - x)$ [where m = total mass = $\rho A\lambda$]

$T = m'\omega^2\left[\frac{\ell - x}{2} + x\right]$

$\Rightarrow \quad T = \frac{m}{\ell}(\ell - x)\omega^2\left(\frac{\ell + x}{2}\right) \qquad T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$





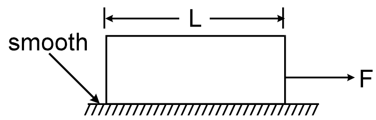
this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' = $\frac{Tdx}{AY}$

Total elongation $\delta = \int \frac{Tdx}{AY} = \int_0^\ell \frac{m\omega^2(\ell^2 - x^2)}{2\ell \quad AY} \; dx$

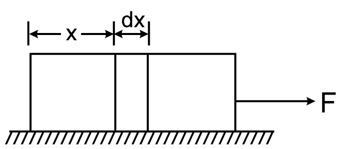
$$\delta = \frac{m\omega^2}{2\ell \quad AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^\ell = \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

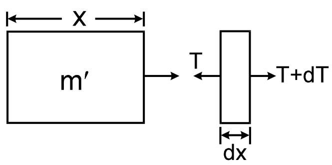
$$\delta = \frac{\rho\omega^2\ell^3}{3Y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}} = 9 \times 10^{-3} \text{ m} = 9\text{mm}$$

Example 5. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and Y respectively.



Solution :





Acceleration, $a = \frac{F}{m}$ then $T = m'a$ where $\Rightarrow$ $m' = \frac{m}{\ell}x$

$T = \frac{m}{\ell} x \frac{F}{m} = \frac{F \; x}{\ell}$

Elongation in element 'dx' = $\frac{Tdx}{AY}$

total elongation, $\delta = \int_0^{\ell} \frac{Tdx}{AY}$ $d = \int_0^{\ell} \frac{Fxdx}{A\ell Y} = \frac{F\ell}{2AY}$

Note :-Try this problem, if friction is given between block and surface (μ = friction coefficient), and

Case : (I) F < μmg (II) F > μmg

Ans. In both cases answer will be $\frac{F\ell}{2AY}$

2. Bulk modulus :

It is defined as the ratio of the normal stress to the volume strain

i.e. $B = \frac{\text{Pressure}}{\text{Volume strain}}$

The stress being the normal force applied per unit area and is equal to the pressure applied (p).

$$B = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in SI is N^{-1} m^2 or $pascal^{-1}$ (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids, and for gases it is 10^{-8} times of solids.

$B_{solids} > B_{liquids} > B_{gases}$

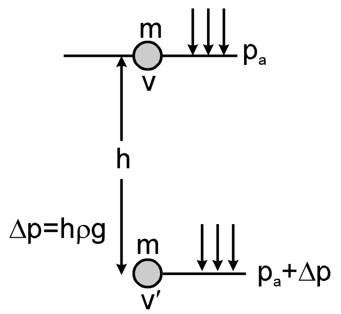
Isothermal bulk modulus of elasticity of gas B = P (pressure of gas)

Adiabatic bulk modulus of elasticity of gas B = γ × P where $\gamma = \frac{C_p}{C_v}$.

Solved Example

Example 6. Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility $K = 50 \times 10^{-6}$ /atm.

Solution : $B = \frac{\Delta p}{-\frac{\Delta V}{V}}$ $\frac{\Delta V}{V} = -\frac{\Delta p}{B}$



m = ρV = const.

$d\rho\, V + dV\,.\,\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$

i.e. $\frac{\Delta\rho}{\rho} = \frac{\Delta p}{B} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100}$

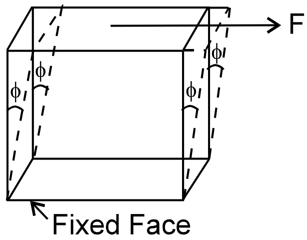
$\frac{1}{100} = \frac{h\rho g}{B}$ [assuming ρ = const.]

$h\rho g = \frac{B}{100} = \frac{1}{100K} \Rightarrow h\rho g = \frac{1\times10^5}{100\times50\times10^{-6}}$

$h = \frac{10^5}{5000\times10^{-6}\times1000\times10} = \frac{100\times10^3}{50} = 2km$ **Ans.**

3. Modulus of Rigidity :

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.



$\therefore$ Tangential stress = F/A.

Let the vertical sides of the cube shifts through an angle θ, called shear strain

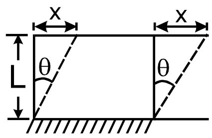
$\therefore$ Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} \quad \text{or} \quad \eta = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

Solved Example

Example 7. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.

Solution : $L = 5 \times 10^{-2}$ m $\Rightarrow \frac{F}{A} = \eta \frac{x}{L}$

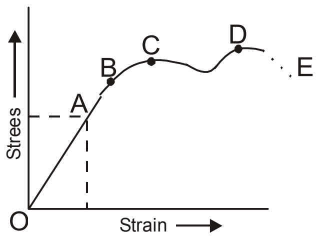


strain $\theta = \frac{F}{A\eta} = \frac{1800}{25 \times 10^{-4} \times 2.4 \times 10^6} = \frac{180}{25 \times 24} = \frac{3}{10} = 0.3$ radian

$\frac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 5 \times 10^{-2} = 1.5 \times 10^{-2}$ m $= 1.5$ cm **Ans.**

VARIATION OF STRAIN WITH STRESS

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. In shown figure, this type of behavior is represented by OB portion of the graph. Till A the stress is proportional to strain and from A to B if deforming forces are removed then the wire comes to its original length but here stress is not proportional to strain.



OA $\rightarrow$ Limit of Proportionality
OB $\rightarrow$ Elastic limit
C $\rightarrow$ Yield Point
CD $\rightarrow$ Plastic behaviour
D $\rightarrow$ Ultimate point
DE $\rightarrow$ Fracture

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behavior is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

Important points

- Breaking stress = Breaking force/area of cross section.
- Breaking stress is constant for a material.
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that safety factor = breaking stress/working stress may have a large value.
- Breaking strain = elongation or compression/original dimension.
- Breaking strain is constant for a material.

Elastic after effect

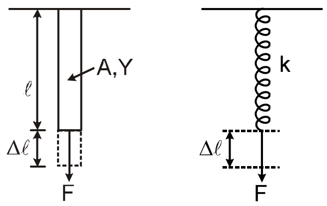
We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz fiber and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers.

For glass fiber elastic after effect is very large. It takes hours for glass fiber to return to its original state on removal of deforming force.

Elastic Fatigue

The, the loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a longtime of their use.

Analogy of Rod as a spring

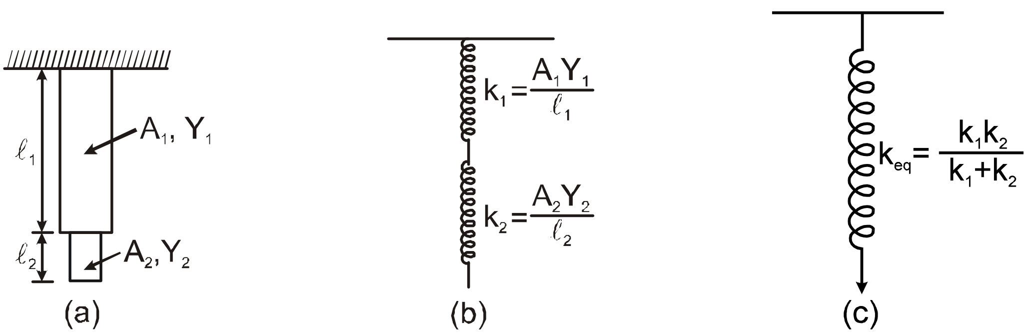


$Y = \frac{\text{stress}}{\text{strain}} \quad \Rightarrow \quad Y = \frac{F\ell}{A\Delta\ell}$

or $F = \frac{AY}{\ell}\Delta\ell$

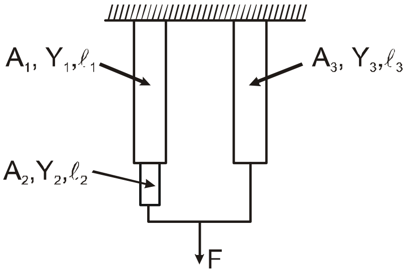
$\frac{AY}{\ell}$ = constant, depends on type of material and geometry of rod. $F = k\Delta\ell$

where $k = \frac{AY}{\ell}$ = equivalent spring constant.

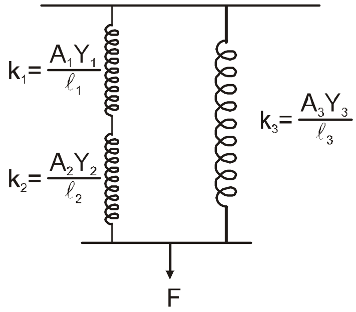


for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.

Figure (d) represents another combination of rods and their replaced spring system.

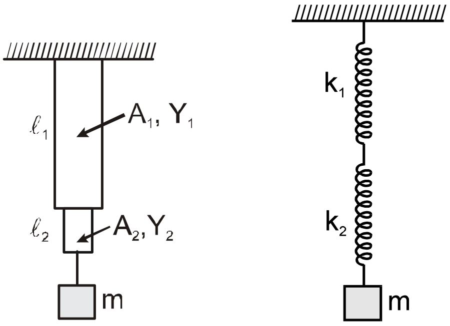


(d)



Solved Example

Example 8. A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.

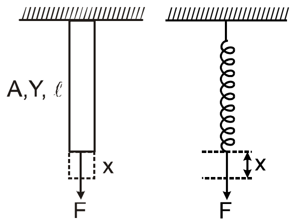


Solution : $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$

$T = 2\pi\sqrt{\dfrac{m}{k_{eq}}} = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

where $k_1 = \dfrac{A_1 Y_1}{\ell_1}$ and $k_2 = \dfrac{A_2 Y_2}{\ell_2}$

ELASTIC POTENTIAL ENERGY STORED IN A STRETCHED WIRE OR IN A ROD



Strain energy stored in equivalent spring

$U = \dfrac{1}{2}kx^2$

where $x = \dfrac{F\ell}{AY}$, $k = \dfrac{AY}{\ell}$

$U = \dfrac{1}{2}\dfrac{AY}{\ell}\dfrac{F^2\ell^2}{A^2Y^2} = \dfrac{1}{2}\dfrac{F^2\ell}{AY}$

equation can be re-arranged

$U = \dfrac{1}{2}\dfrac{F^2}{A^2} \times \dfrac{\ell A}{Y}$ [ℓA = volume of rod, F/A = stress]

$U = \dfrac{1}{2}\dfrac{(\text{stress})^2}{Y} \times \text{volume}$

again, $U = \dfrac{1}{2}\dfrac{F}{A} \times \dfrac{F}{AY} \times A\,\ell$ [Strain = $\dfrac{F}{AY}$]

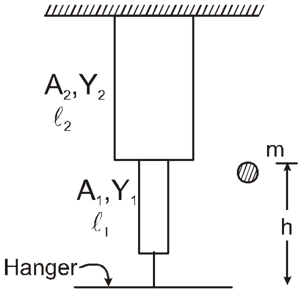
$$U = \frac{1}{2} \text{stress} \times \text{strain} \times \text{volume}$$

again, $U = \frac{1}{2}\frac{F^2}{A^2Y^2}A\ell Y \quad \Rightarrow \quad U = \frac{1}{2}Y\,(\text{strain})^2 \times \text{volume}$

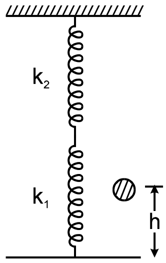
strain energy density $= \frac{\text{strain energy}}{\text{volume}} = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2}\text{stress} \times \text{strain}$

Solved Example

Example 9. A ball of mass 'm' drops from a height 'h', which sticks tomass-less hanger after striking. Neglect over turning, find out the maximum extension in rod. Assuming rod is massless.



Solution : Applying energy conservation $mg\,(h + x) = \frac{1}{2}\frac{k_1k_2}{k_1+k_2}x^2$



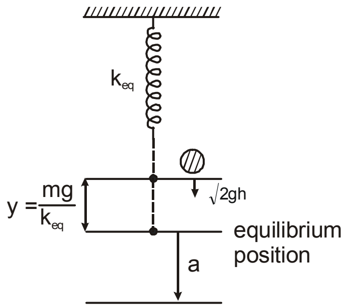
where $k_1 = \frac{A_1Y_1}{\ell_1} \qquad k_2 = \frac{A_2Y_2}{\ell_2}$

& $k_{eq} = \frac{A_1A_2Y_1Y_2}{A_1Y_1\ell_2 + A_2Y_2\ell_1}$

$k_{eq}x^2 - 2mgx - 2mgh = 0$

$x = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk_{eq}}}{2k_{eq}}$, $x_{max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$

OTHERWAY BY S.H.M.



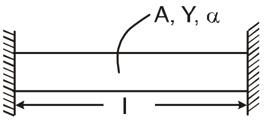
$\omega = \sqrt{\frac{k_{eq}}{m}} \qquad v = \omega\sqrt{a^2 - y^2}$

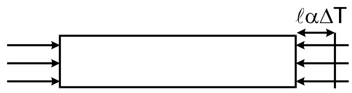
$\sqrt{2gh} = \sqrt{\frac{k_{eq}}{m}}\sqrt{a^2 - y^2} \quad \Rightarrow \quad \sqrt{\frac{2mgh}{k_{eq}} + \frac{m^2g^2}{k_{eq}^2}} = a$

$\max^m$ extension

$= a + y = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}} + \frac{2mgh}{k_{eq}}}$

THERMAL STRESS :



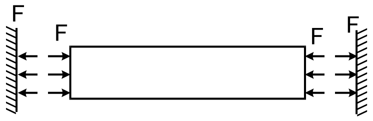


If temp of rod is increased by ΔT, then change in length $\Delta \ell = \ell \, \alpha \, \Delta T$

strain = $\frac{\Delta \ell}{\ell} = \alpha \quad \Delta T$

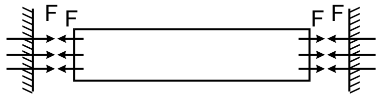
But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

$Y = \frac{\text{stress}}{\text{strain}}$



If ΔT = positive

thermal stress = Y strain = $Y \, \alpha \, \Delta T$

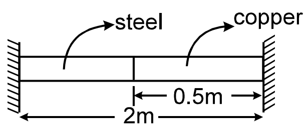


If ΔT = negative

$\frac{F}{A} = Y\alpha\Delta T \qquad F = AY \, \alpha \, \Delta T$

Solved Example

Example 10. When composite rod is free, then composite length increases to 2.002 m for temperature rise from 20ºC to 120ºC. When composite rod is fixed between the support, there is no change in component length find Y and α of steel, if $Y_{cu} = 1.5 \times 10^{13}$ N/m^2 $\alpha_{cu} = 1.6 \times 10^{-5}$/ºC.



Solution : $\Delta \ell = \ell_s \, \alpha_s \, \Delta T + \ell_c \, \alpha_c \Delta T$

$.002 = [1.5 \, \alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$

$\alpha_s = \frac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}$/ºC

there is no change in component length

For steel

$x = \ell_s \, \alpha_s \, \Delta T - \frac{F\ell_s}{AY_s} = 0$

$\frac{F}{AY_s} = \alpha_s \quad \Delta T$ ....(A)

for copper

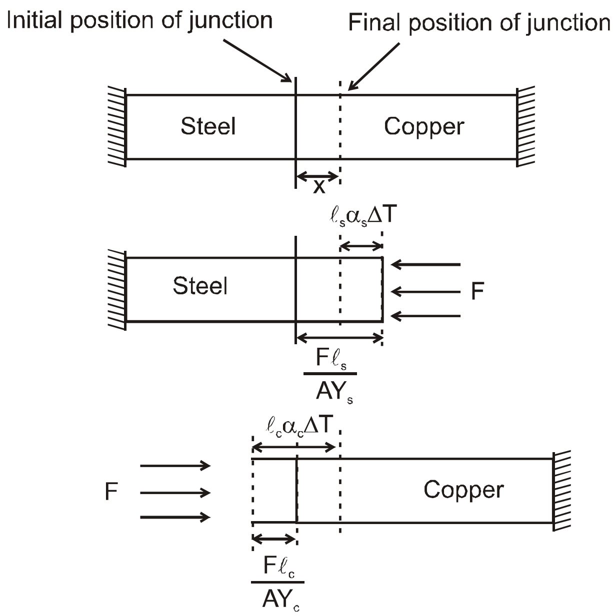
$x = \frac{F\ell_c}{AY_c} - \ell_c \, \alpha_c \, \Delta T = 0$

$\frac{F}{AY_c} = \alpha_c \Delta T$ ...(B)

B/A $\Rightarrow \frac{Y_s}{Y_c} = \frac{\alpha_c}{\alpha_s}$

$Y_s = Y_c \frac{\alpha_c}{\alpha_s} = \frac{1.5 \times 10^{13} \times 16 \times 10^{6}}{8 \times 10^{-6}}$

$= 3 \times 10^{13}$ N/m^2



APPLICATIONS OF ELASTICITY

Some of the important applications of the elasticity of the materials are discussed as follows :

1. The material used in bridges lose its elastic strength with time bridges are declared unsafe after long use.

2. To estimate the maximum height of a mountain :

The pressure at the base of the mountain = $h\rho g$ = stress. The elastic limit of a typical rock is 3×10^8 N m^{-2}

The stress must be less than the elastic limits, otherwise the rock begins to flow.

$h < \frac{3\times10^8}{\rho g} \Rightarrow h < 10^4$ m ($\because \rho = 3 \times 10^3$ kg m^{-3} ; g = 10 ms^{-2}) or h = 10 km

It may be noted that the height of Mount Everest is nearly 9 km.

TORSION CONSTANT OF A WIRE

$C = \frac{\pi\eta r^4}{2\ell}$ Where η is modulus of rigidity r and ℓ is radius and length of wire respectively.

(a) Toque required for twisting by an angle θ, $\tau = C\theta$.

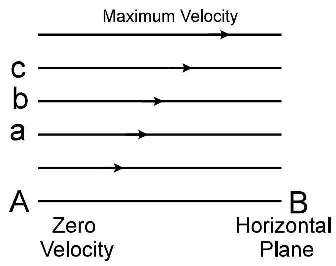
(b) Work done in twisting by an angle θ, $W = \frac{1}{2} C\theta^2$.

VISCOSITY

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c. Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**

The property of viscosity is seen in the following examples :



(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerin, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

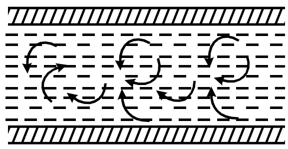
(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

(iv) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

FLOW OF LIQUID IN A TUBE: CRITICAL VELOCITY

When a liquid flows in a tube, the viscous forces oppose the flow of the liquid, Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag and eddy-currents are developed in it.



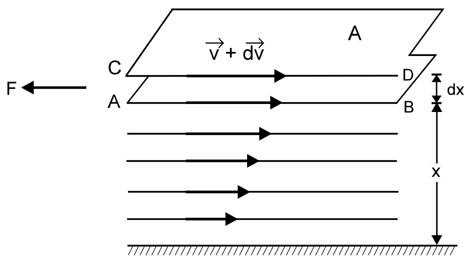
Reynold's proved that the critical velocity for a liquid flowing in a tube is $v_c = k\eta/\rho r$. where ρ is density and η is viscosity of the liquid, r is radius of the tube and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity, the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedily inspite of being very thick (of large viscosity).

VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

The property of a liquid by virtue of which an opposing force (internal friction) comes into play when ever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + d\vec{v}$ at a distance x and (x + dx) respectively from the fixed solid surface. According to Newton, the viscous drag or back ward force (F) between these layers depends.

(i) directly proportional to the area (A) of the layer and

(ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.



i.e. $F \propto A\frac{dv}{dx}$ or $F = -\eta A\frac{dv}{dx}$ ...(1)

η is called Coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.

2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.

3. Both are due to molecular attractions.

Differences between them →

Viscosity	Solid Friction
(i) Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i) Friction between two solids is independent of the area of solid surfaces in contact.
(ii) Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii) Friction is independent of the relative velocity between two surfaces.
(iii) Viscous drag is independent of normal reaction between two layers of liquid.	(iii) Friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.

2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.

4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.

5. It finds an important use in the circulation of blood through arteries and veins of human body.

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have $\eta = \dfrac{F}{A(\Delta v_x / \Delta z)}$

$\therefore$ dimensions of $\eta = \dfrac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \dfrac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$

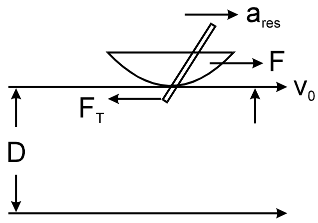
Its unit is kg/(meter-second)*

In C.G.S. system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N sm^{-2} and is called decapoise.

1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

Solved Example

Example 11. A man is rowing a boat with a constant velocity 'v_0' in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.



Solution : $F - F_T = m\, a_{res}$

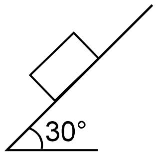
As boat moves with constant velocity $a_{res} = 0$

$F = F_T$

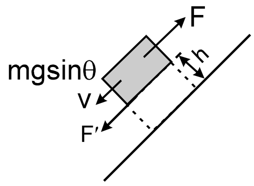
But $F_T = \eta A \frac{dv}{dz}$, but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$

then $F = F_T = \frac{\eta A v_0}{D}$

Example 12. A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. (g = 10 m/sec^2) find out the thickness of layer of liquid.



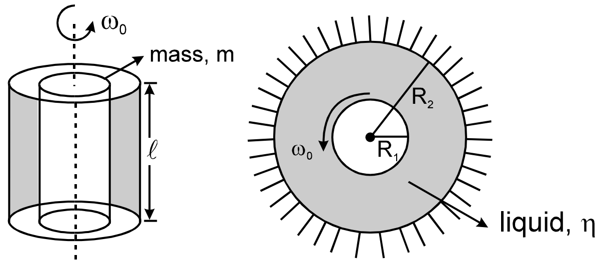
Solution : $F = \eta A \frac{dv}{dz} = mg \sin\theta \quad \frac{dv}{dz} = \frac{v}{h}$



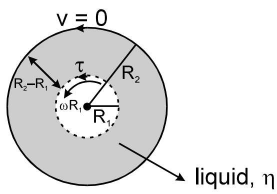
$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$

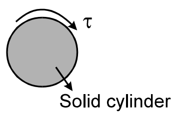
$h = \frac{40 \times 10^{-2}}{100}$ [$\eta = 10^{-1}$ poise $= 10^{-2}$ N-sec-m^{-2}] $= 4 \times 10^{-3}$ m = 4 mm

Example 13. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.



Solution : $F = \eta A \frac{dv}{dz}$, where $\frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$





$F = \eta \frac{2\pi \; R_1 \; \ell \; \omega \; R_1}{R_2 - R_1}$

and $\quad \tau = FR_1 = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$I\alpha = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$\Rightarrow \frac{mR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$ or $-\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \int_0^t dt$

$\Rightarrow t = \frac{m(R_2 - R_1)\ell n 2}{4\pi\eta\ell R_1}$

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$ On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$

STOKE'S LAW

Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity η is given by $F = 6\,\pi\,\eta\,r\,v$. This is called Stokes' law.

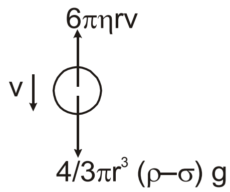
TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward = $V(\rho - \sigma)\,g = \frac{4}{3}\pi r^3 (\rho - \sigma)g$,



(ii) viscous force acting upward = $6\,\pi\,\eta\,rv$.
Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi rv = p\, r^3 (\rho - \sigma)\, g \quad \text{or} \quad v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Important point

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Solved Example

Example 14. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Solution : Rate of heat loss = power = $F \times v = 6\,\pi\,\eta\,r\,v \times v = 6\,\pi\,\eta\,r\,v^2 = 6p\,\eta\,r\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$

Rate of heat loss α r^5

Example 15. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg /(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Solution : By Stoke's law , the terminal velocity of a water drop of radius r is given by $\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values :

$$\nu = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2\times(1.0\times10^3)\times9.8}{1.8\times10^{-5}} = \mathbf{2.72 \times 10^{-4}\ m/s}$$

Example 16. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given : coefficient of viscosity of water = 1.0×10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0×10^3 kg/m^3.

Solution : The velocity attained by the sphere in falling freely from a height h is

$$\nu = \sqrt{2gh} \qquad ....(i)$$

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere σ (= 1.0×10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad \nu = \frac{2\times(1.0\times10^{-3})^2(1.0\times10^4-1.0\times10^3)\times10}{9\times1.0\times10^{-3}} = 20 \text{ m/s}$$

from equation (i), we have $h = \frac{\nu^2}{2g} = \frac{20\times20}{2\times10} = 20$ m

Applications of Stokes' Formula

(i) In determining the Electronic Charge by Millikan's Experiment : Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops : Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity,

(iii) Parachute : When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air. In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. In Fig graph is shown between the speed of the falling soldier and time.